Test Review

(1) (a) $9^{-1} = \frac{1}{9}$ (b) $9^{-2} = \left(\frac{1}{9}\right)^2 = \frac{1}{81}$

(c) $9^0 = 1$ (d) $9^3 = 729$ $\begin{array}{r} 81 \\ 9 \\ \hline 729 \end{array}$

(e) $9^{\frac{1}{2}} = \sqrt{9} = 3$ (f) $9^{-\frac{1}{2}} = \frac{1}{9^{1/2}} = \frac{1}{3}$

(g) $9^{3/2} = 3^3 = 27$ (h) $9^{-3/2} = \frac{1}{9^{3/2}} = \frac{1}{27}$

(i) $\left(\frac{8}{125}\right)^{1/3} = \sqrt[3]{\frac{8}{125}} = \frac{2}{5}$ (j) $\left(\frac{8}{125}\right)^{-1/3} = \frac{5}{2}$

(k) $\left(\frac{8}{125}\right)^{2/3} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$ (l) $\left(\frac{8}{125}\right)^{-2/3} = \frac{25}{4}$

(2) (a) $4^3 = 64 \Leftrightarrow \log_4 64 = 3$

(b) $2^5 = 32 \Leftrightarrow \log_2 32 = 5$

(c) $e^{1/2} = \sqrt{e} \Leftrightarrow \ln \sqrt{e} = 1/2$

(d) $e^0 = 1 \Leftrightarrow \ln 1 = 0$

(3) (a) $\log_3 27 = 3 \iff 3^3 = 27$

(b) $\log_{10}\sqrt{10} = \frac{1}{2} \iff 10^{1/2} = \sqrt{10}$

(c) $\ln e^2 = 2 \iff e^2 = e^2$

(d) $\ln \frac{1}{e} = -1 \iff e^{-1} = \frac{1}{e}$

(4) (a) $\log_2 4 = 2$

(b) $\log_4 2 = \frac{1}{2}$

because $4^{1/2} = 2$

(c) $\log_5 1 = 0$

(d) $\log_2 \frac{1}{2} = -1$

because $2^{-1} = 1/2$

(e) $\log_{10}\sqrt{10} = 1/2$

because $10^{1/2} = \sqrt{10}$

(f) $\log_6 \frac{1}{\sqrt{6}} = -1/2$

because $6^{-1/2} = \frac{1}{\sqrt{6}}$

(g) $\log_8 8 = 1$

(h) $\log_8 4 = 2/3$

$8^x = 4$

$(2^3)^x = 2^2 \rightarrow 3x = 2$

$x = 2/3$

4(i) $\log_3 81 = 4$

because $3^4 = 81$

(j) $\log_3 \frac{1}{3} = -1$

because $3^{-1} = \frac{1}{3}$

(k) $\log_3 3 = 1$

(l) $\log_3 1 = 0$

(m) $\log_3 \sqrt[3]{3} = \frac{1}{3}$

because $3^{1/3} = \sqrt[3]{3}$

(n) $\log_3 \frac{1}{\sqrt{3}} = -\frac{1}{2}$

because $3^{-1/2} = \frac{1}{\sqrt{3}}$

(o) $\log_3 \frac{1}{9} = -2$

because $3^{-2} = 9^{-1} = \frac{1}{9}$

(p) $\log_3 3^5 = 5$

because $3^5 = 3^5$ (!)

(q) $\ln e = 1$

because $e^1 = e$

(r) $\ln \frac{1}{e} = -1$

because $e^{-1} = \frac{1}{e}$

(s) $\ln e^3 = 3$

(t) $\ln \sqrt{e} = 1/2$

(5) (a) $\log_2 3 + \log_2 y = \log_2(3y)$

(b) $2\log_2 5 + 3\log_2 x = \log_2 5^2 + \log_2 x^2$

$= \log_2(25x^2)$

(c) $\log_3 p - \log_3 q = \log_3 \frac{p}{q}$

(d) $3\log_3 x - 4\log_3 y = \log_3 x^3 - \log_3 y^4$

$= \log_3\left(\frac{x^3}{y^4}\right)$

(e) $\ln x - \ln 5 = \ln\left(\frac{x}{5}\right)$

(f) $2\ln 4 - 3\ln y = \ln 4^2 - \ln y^3$

$= \ln\left(\frac{16}{y^3}\right)$

(6) (a) $\log_2 4x = \log_2 4 + \log_2 x = 2 + \log_2 x$

(b) $\log_2 \frac{p}{q} = \log_2 p - \log_2 q$

6(c) $\log_3 4x^3 = \log_3 4 + \log_3 x^3$

$= \log_3 4 + 3 \cdot \log_3 x$

(d) $\log_3\left(\frac{x^4}{y^6}\right) = \log_3 x^4 - \log_3 y^6$

$= 4 \cdot \log_3 x - 6 \cdot \log_3 y$

(e) $\ln \frac{x}{5yz} = \ln x - \ln 5 - \ln y - \ln z$

(f) $\ln\left(\frac{w x^2}{y^3 z^4}\right) = \ln w + 2\ln x - 3\ln y - 4\ln z$

(7) (a) $\log_2 5 + \log_2 x = 3$

condense: $\log_2 (5x) = 3$

write in exponential form: $2^3 = 5x$

solve: $\boxed{x = \frac{8}{5}}$

$\log_2 \boxed{16} = 4$

$2^4 = 16$

7(b) $\underline{\log_3 x} + \underline{\log_3 (x-2)} = 1$

condense: $\log_3 (x^2 - 2x) = 1$

exponential form: $3^1 = x^2 - 2x$

$$3 = x^2 - 2x \Longrightarrow x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$$\cancel{x = -1} \text{ or } \boxed{x = 3}$$

(c) $\ln x + \ln 3 = 1$

$$\ln x = 1 - \ln 3$$

$$\boxed{x = e^{1-\ln 3}}$$

OR

$$\ln x + \ln 3 = 1$$

$$\ln(3x) = 1$$

$$e^1 = 3x$$

$$\boxed{x = \frac{1}{3}e}$$

(d) $2^{5x-1} = 4$ alternate way to solve on the next page

$$\ln 2^{5x-1} = \ln 4$$

$$(5x-1)\ln 2 = \ln 4$$

$$5x \ln 2 - \ln 2 = \ln 4$$

$$5x \ln 2 = \ln 4 + \ln 2$$

$$x = \frac{\ln 4 + \ln 2}{5 \cdot \ln 2} = \frac{\ln 8}{5 \cdot \ln 2}$$

$$= \frac{\ln 2^3}{5 \cdot \ln 2} = \frac{3 \cdot \cancel{\ln 2}}{5 \cdot \cancel{\ln 2}} = \frac{3}{5}$$

<u>OR</u> (7d revisited)

$$2^{5x-1} = 4$$

$$2^{5x-1} = 2^2 \Rightarrow 5x - 1 = 2$$

$$5x = 3$$

$$\boxed{x = \frac{3}{5}}$$

7(e) $5^{x^2} = 25$

$$5^{x^2} = 5^2$$

$$x^2 = 2$$

$$\boxed{x = \pm\sqrt{2}}$$

<u>OR</u>

$$\ln 5^{x^2} = \ln 25$$

$$x^2 \ln 5 = \ln 25$$

$$x^2 = \frac{\ln 25}{\ln 5} = \frac{\ln 5^2}{\ln 5}$$

$$x^2 = \frac{2\cancel{\ln 5}}{\cancel{\ln 5}} = 2$$

$$\boxed{x = \pm\sqrt{2}}$$

7(f) $9^x = 27^{5x+2}$

$(3^2)^x = (3^3)^{5x+2}$

$3^{2x} = 3^{15x+6}$

$2x = 15x + 6$

$-6 = 13x$

$\boxed{-\frac{6}{13} = x}$

7(g) $5^{2x} = 6^{x-3}$

$\ln 5^{2x} = \ln 6^{x-3}$

$2x \cdot \ln 5 = (x-3) \cdot \ln 6$

$2x \cdot \ln 5 = x \cdot \ln 6 - 3 \cdot \ln 6$

$2x \ln 5 - x \cdot \ln 6 = -3 \cdot \ln 6$

$x(2 \cdot \ln 5 - \ln 6) = -3 \cdot \ln 6$

$\boxed{x = \frac{-3 \ln 6}{2 \ln 5 - \ln 6}}$

7(h) $e^{2x+3} = 10$

$$\ln e^{2x+3} = \ln 10$$

$$(2x+3)\cdot \cancel{\ln e}^{1} = \ln 10$$

$$2x = -3 + \ln 10$$

$$\boxed{x = \frac{-3 + \ln 10}{2}}$$

(8) First, find an equation for mass in terms of time: $m(t) = a \cdot b^t$

Since an initial quantity isn't specified, you can use any quantity you choose. I am going to use 1.000 g for the initial quantity.

$$m(t) = 1 \cdot b^t$$

It will take 3.8235 days to decay to half the initial quantity.

$$0.5 = 1 \cdot b^{3.8235}$$

$$(0.5)^{\frac{1}{3.8235}} = \left(b^{3.8235}\right)^{\frac{1}{3.8235}}$$

$$0.83420 = b$$

$$\boxed{m(t) = 0.83420^t}$$

How long until only 10% remains?

$$0.1 = 0.83420^t$$

$$\ln(0.1) = t \cdot \ln(0.83420)$$

$$t = \frac{\ln 0.1}{\ln 0.83420} \approx 12.7 \text{ days}$$

(9) First, write an equation.

$m(t) = 100 \cdot b^t$

$61.57 = 100 \cdot b^{10}$ ← After 10 days

$0.6157 = b^{10}$

$(0.6157)^{\frac{1}{10}} = (b^{10})^{1/10}$

$0.95266 = b$

$\boxed{m(t) = 100(0.95266)^t}$

(c) Find decay rate

$b = 1 + r$

$0.95266 = 1 + r$

$-0.04734 = r$

decay rate: 4.73% per day

(a) After 1 half-life, 50 g remain:

$50 = 100(0.95266)^t$

$1/2 = 0.95266^t$

$\ln \frac{1}{2} = t \cdot \ln 0.95266$

$t = \frac{\ln 1/2}{\ln 0.95266} = 14.3$ days

(b) $1 = 100(0.95266)^t$

$0.01 = 0.95266^t$

$\ln(0.01) = t \cdot \ln 0.95266$

$$t = \frac{\ln 0.01}{\ln 0.95266} = 95.0 \text{ days}$$

$$\boxed{b = 1 + r}$$

(10) $$\boxed{P(t) = \overset{a}{12342}\;\overset{b^t}{(1+0.0213)^t}}$$

(a) $$P(4) = 12342(1.0213)^4$$

$$= 13428 \quad \underline{\text{or}} \quad 13400$$

(b) $$15000 = 12342(1.0213)^t$$

$$1.21586 = 1.0213^t$$

$$\ln 1.21586 = t \cdot \ln 1.0213$$

$$\frac{\ln 1.21586}{\ln 1.0213} = t = 9.25 \text{ yrs}$$

(11) $t = 0 \longleftrightarrow$ 5 yrs ago

$t = 5 \longleftrightarrow$ now

$$P(t) = a \cdot b^t$$

$$P(t) = 542900 \cdot b^t$$

plug in $\left(\overset{t}{5}, \overset{P}{524700} \right)$

$$\frac{524700}{542900} = \frac{542900\, b^5}{542900}$$

$$(0.96648)^{\frac{1}{5}} = (b^5)^{\frac{1}{5}}$$

$$0.99320 = b$$

$$\boxed{P(t) = 542900(0.99320)^t}$$

(a) $b = 1 + r \rightarrow 0.99320 = 1 + r$

$r = -0.00680$

rate of decline : 0.680% per year

(b) $P(14) = \underline{542900(0.99320)^{14}}$

9 yrs from now: $t = 5 + 9 = 14$

$P(14) = 493439$ or 493000

(c) $\dfrac{500000}{542900} = \dfrac{542900(0.99320)^{t}}{542900}$

$0.92098 = 0.99320^{t}$

$\ln 0.92098 = t \cdot \ln 0.99320$

$\dfrac{\ln 0.92098}{\ln 0.99320} = t = 12.1$

the pop. will reach half a million 7.06 ~~12.1~~ years from now.