Logarithms

$3^4 = 81$

exactly the same

$\log_3 81 = 4$

argument of the log

logarithm (exponent)

Ex. $\log_4 2 = \boxed{1/2}$

$4^{\boxed{1/2}} = 2$

Ex. $\log_3 9 = \boxed{2}$

$3^{\boxed{2}} = 9$

Ex. $\log_3 \sqrt{3} = \boxed{1/2}$

$$3^{\boxed{1/2}} = \sqrt{3}$$

Ex. $\log_{10} (0.0001) = \boxed{}$

$$10^{\boxed{-4}} = \frac{1}{10000} = \frac{1}{10^4} = 10^{-4}$$

#2

(a) 2 (b) $\frac{1}{2}$ (c) 3 (d) -1

(e) -2 (f) $\frac{1}{2}$ (g) $-\frac{1}{2}$ (h) 0

(i) 1 (j) 4 (k) 10 (l) -4

$$\boxed{\log_3\left(\frac{1}{3^4}\right) = -4}$$

$$3^{\boxed{-4}} = \frac{1}{3^4} = 3^{-4}$$

m) $\frac{1}{3}$ n) $-\frac{1}{3}$ o) 2 p) 3

$$\sqrt[3]{2} = 2^{1/3} \qquad \sqrt[4]{2} = 2^{1/4}$$

$$\sqrt[n]{p} = p^{1/n} \qquad 25^{3/2} = \left(\sqrt{25}\right)^3 = 125$$

$$\frac{1}{\sqrt[3]{2}} = 2^{-1/3}$$

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q) 4 r) 7 s) -3 t) $-\frac{1}{2}$

u) 5 v) -5 w) 25 x) -1

Exponential : $b^L = a$ inter-changable

Logarithm : $\log_b a = L$

$2^{11} = 2048 \leftarrow$ exponential form

$\log_2 2048 = 11 \leftarrow$ log form

#3 (a) $5^3 = 125$

$\log_5 125 = 3$

#3 (b) $\log_2 1024 = 10$

(c) $\log_3 \sqrt{3} = \frac{1}{2}$

(d) $\log_{27} 9 = \frac{2}{3}$

(e) $\log_{10} 1 = 0$

$\boxed{\log 1 = 0}$

(f) $\log_8 8 = 1$

#4 (a) $3^2 = 9$ (b) $10^{-2} = \frac{1}{100}$

(c) $5^{1/3} = \sqrt[3]{5}$ (d) $16^0 = 1$

The Log Rules

Recall: $\boxed{a^m \cdot a^n = a^{m+n}}$

$$2^3 \cdot 2^5 = 2^{3+5} = 2^8$$

$$\underbrace{(2\cdot 2\cdot 2)\cdot(2\cdot 2\cdot 2\cdot 2\cdot 2)}$$

Ex. $2^{\log_2 4} \cdot 2^{\log_2 8} = 2^{\log_2 4 + \log 8}$

$2^2 \cdot 2^3 \qquad = 2^{\log_2 32}$

$4 \cdot 8$

32

Log Rule #1: $\boxed{\log a + \log b = \log(ab)}$

Log Rule #2: $\boxed{\log a - \log b = \log \frac{a}{b}}$

Log Rule #3: $\boxed{\log a^b = b \cdot \log a}$

Expand: $\log_2(32 \cdot x^2 \cdot y^3)$ rule 1
(eliminate exponents)

$= \log_2 32 + \log_2 x^2 + \log_2 y^3$ rule 3

$= 5 + 2 \cdot \log_2 x + 3 \cdot \log_2 y$

Expand: $\log_{10}\left(\frac{100}{x^3 y^4}\right)$ rule#2

$= \log_{10} 100 - \underbrace{\log_{10}(x^3 y^4)}$

$= \log_{10} 100 - \left(\log_{10} x^3 + \log_{10} y^4\right)$

$= 2 - 3\cdot\log_{10} x - 4\cdot\log_{10} y$

Condense: $\log_3 x - 4\log_3 y$ rule 3

$= \log_3 x - \log_3 y^4$ rule 2

$= \log_3\left(\frac{x}{y^4}\right)$

Using Condensing to solve an equation

Ex. $\log_2 x + \log_2(x-4) = 5$

condense the left side (use rule 1)

$$\log_2[x\cdot(x-4)] = 5$$

log: $\log_2(x^2-4x) = 5$

exponential: $2^5 = x^2-4x$

$$x^2-4x-32=0$$

$$(x+4)(x-8)=0$$

$\cancel{x=-4}$ or $\boxed{x=8}$

HW: Logs #5, 6, 7efg