Trigonometry

$$\text{sine} = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

$$\text{cosine} = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

$$\text{tangent} = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

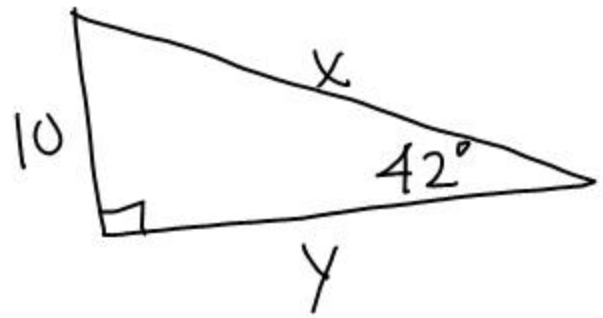
} For right triangles

SOH CAH TOA

Right Triangle Trig

(1) Given a side and an acute angle, find the other sides

Ex.



opp leg ↓

$$\sin 42^\circ = \frac{10}{X}$$

↑ hyp

$$X \cdot \sin 42^\circ = 10$$

$$X = \frac{10}{\sin 42^\circ}$$

$$X \approx 14.9$$

$$\tan 42^\circ = \frac{10}{y}$$ ← opp leg, ← adj leg

$$y \cdot \tan 42^\circ = 10$$

$$y = \frac{10}{\tan 42^\circ} \approx 11.1$$

Ex

13 ← hypotenuse

$$\cos 70^\circ = \frac{a}{13}$$

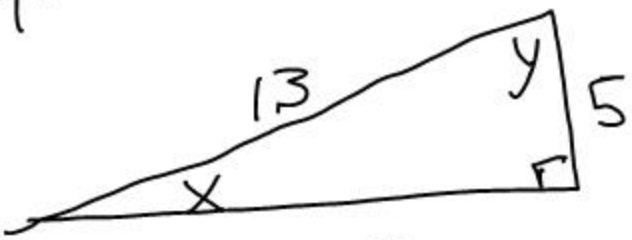
$$a = 13 \cos 70^\circ = 4.4$$

$$\sin 70^\circ = \frac{b}{13}$$

$$b = 13 \sin 70^\circ \approx 12.2$$

(2) Given 2 sides, find the angles

Ex.



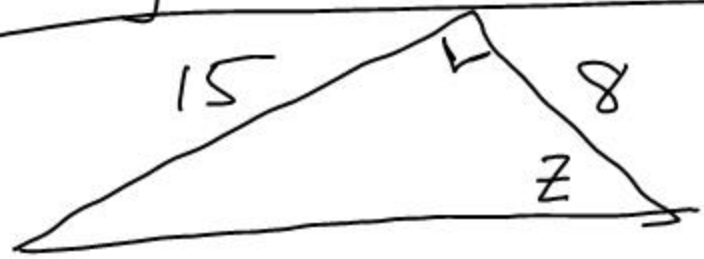
$$\sin x = \frac{5}{13}$$

$$x = \sin^{-1}\left(\frac{5}{13}\right) \approx 22.6^\circ$$

↑ inverse sine

$$y = 90 - 22.6 = 67.4^\circ$$

Ex.



$$\tan z = \frac{15}{8}$$

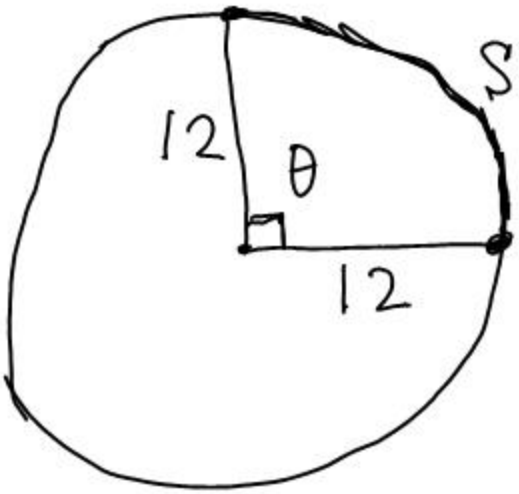
$$z = \tan^{-1}\left(\frac{15}{8}\right) \approx 61.9^\circ$$

Radian Measure

s (arc length)

The radian measure of θ is

$$\theta = \frac{s}{r}$$



$$s = \frac{1}{4}(2\pi \cdot 12) = 6\pi$$

$$\theta = \frac{6\pi}{12} = \frac{\pi}{2}$$

$$90^\circ = \frac{\pi}{2}$$

$$180^\circ = \pi$$

Conversion factors

$$\frac{180^\circ}{\pi} \text{ or } \frac{\pi}{180^\circ}$$

Ex: Convert 15° to radians

$$15^{\cancel{\circ}} \cdot \frac{\pi}{180^{\cancel{\circ}}} = \frac{\pi}{12}$$

Ex. Convert $\frac{3\pi}{4}$ to degrees

$$\frac{3\cancel{\pi}}{4} \cdot \frac{180^{\circ}}{\cancel{\pi}} = 3(45) = 135^{\circ}$$

Some radian measures to memorize

degrees	radians
0°	0
30°	$\pi/6$
45°	$\pi/4$
60°	$\pi/3$
90°	$\pi/2$
120°	$2\pi/3$
135°	$3\pi/4$
150°	$\frac{5\pi}{6}$

180°	π

The "missing" trig functions

- cotangent is the reciprocal of tangent

$$\cot = \frac{\text{adj leg}}{\text{opp leg}}$$

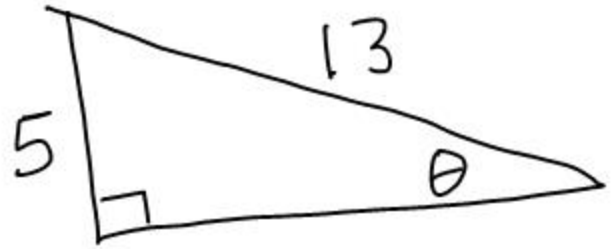
- secant is the reciprocal of cosine

$$\sec = \frac{\text{hyp}}{\text{adj. leg}}$$

- cosecant is the reciprocal of sine

$$\csc = \frac{\text{hyp}}{\text{opp. leg}}$$

Ex.



$13^2 - 5^2$

$= 144$

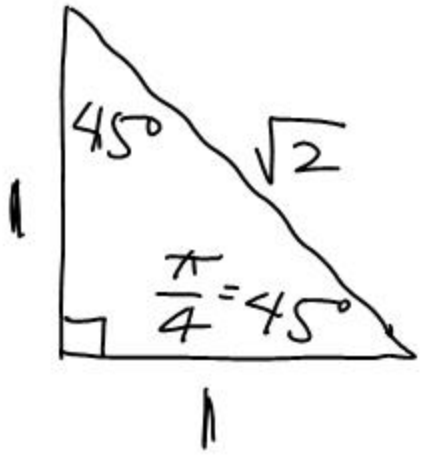
$\sin\theta = \frac{5}{13}$ $\quad \csc\theta = \frac{13}{5}$

$\tan\theta = \frac{5}{12}$ $\quad \sec\theta = \frac{13}{12}$

$\cot\theta = \frac{12}{5}$ $\quad \cos\theta = \frac{12}{13}$

The Special Right Triangles

$45°-45°-90°$



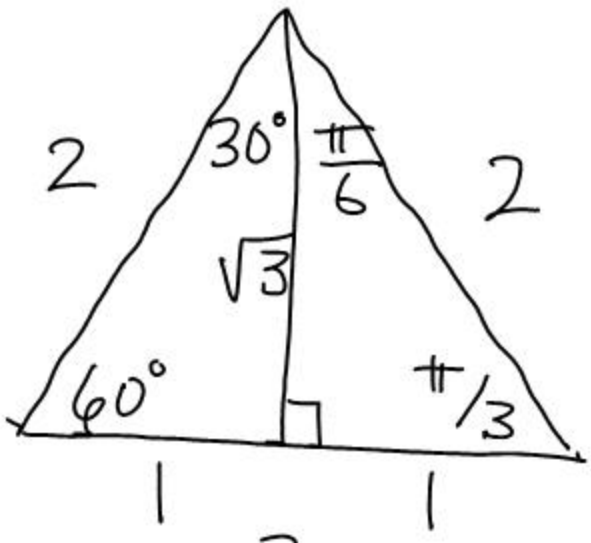
$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\tan\frac{\pi}{4} = \frac{1}{1} = 1$$

Know from memory

$30°-60°-90°$



$$\sin 30° = \frac{1}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

$$\tan\frac{\pi}{3} = \frac{\sqrt{3}}{1} = \sqrt{3}$$