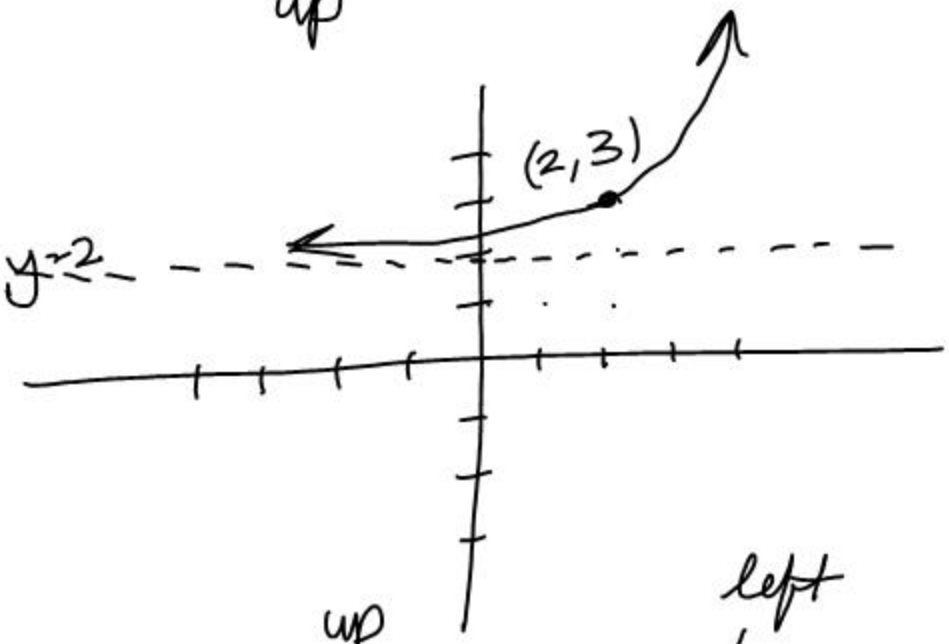
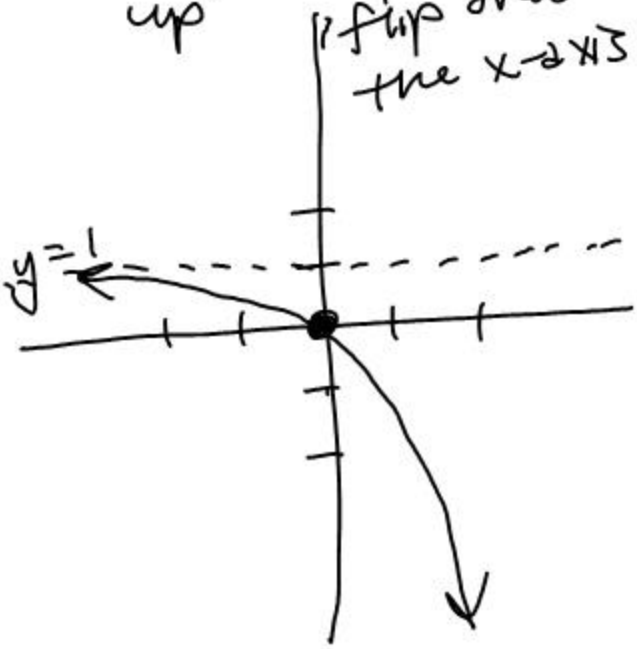
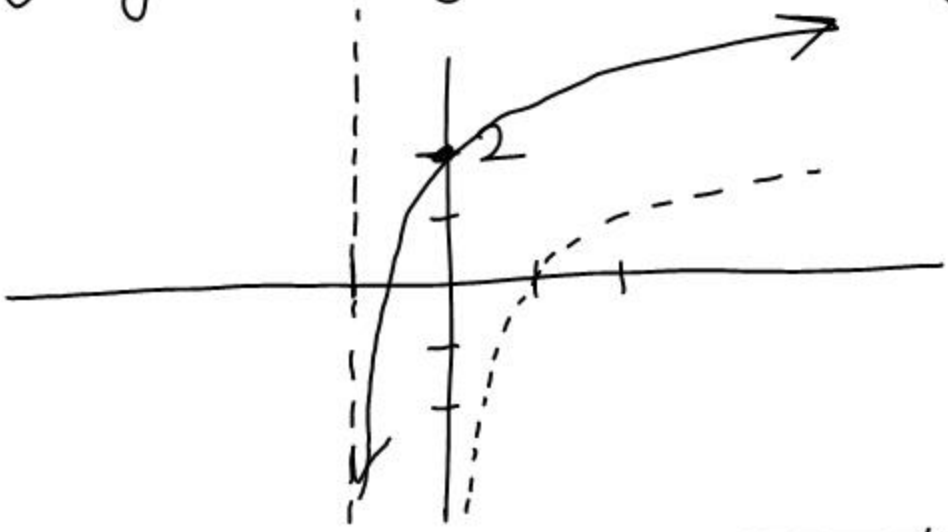
(a) $y = 2 + 2^{x-2}$ — up, right



(b) $y = 1 - e^{x}$ — up, flip over the x-axis



(c) $y = 2 + \log_2(x+1)$ — up, left



(2) (a) $\log_4 64 = 3$

(b) $\log_2 32 = 5$

(c) $\ln \sqrt{e} = \frac{1}{2}$

(d) $\ln 1 = 0$

same thing: $b^L = a$ ⇔ $\log_b a = L$

③ (a) $3^3 = 27$ (b) $10^{1/2} = \sqrt{10}$ (c) $e^2 = e^2$

(d) $e^{-1} = \frac{1}{e}$

④ (a) 2 (b) $\frac{1}{2}$ (c) 0 (d) -1

(e) $\frac{1}{2}$ (f) $-\frac{1}{2}$ (g) 1 (h) $\frac{2}{3}$

(i) 4 (j) -1 (k) 1 (l) 0

(m) $\frac{1}{3}$ (n) $-\frac{1}{2}$ (o) -2 (p) 5

(q) 1 (r) -1 (s) 3 (t) $\frac{1}{2}$

4h $\log_8 4 = x$

$8^x = 4$

$(2^3)^x = 2^2$

$2^{3x} = 2^2$

$3x = 2$

$x = 2/3$

4o $\log_3 \frac{1}{9} = x$

$3^x = \frac{1}{9}$

$3^x = \frac{1}{3^2}$

$3^x = 3^{-2}$

$x = -2$

TEST on Wednesday 11/15

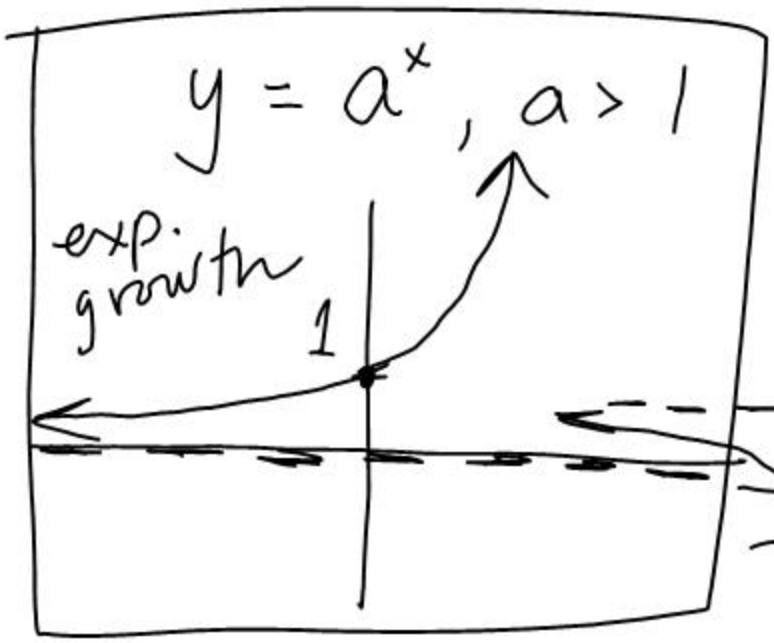
4p $\log_3 3^5 = x$

$3^x = 3^5$

$x = 5$

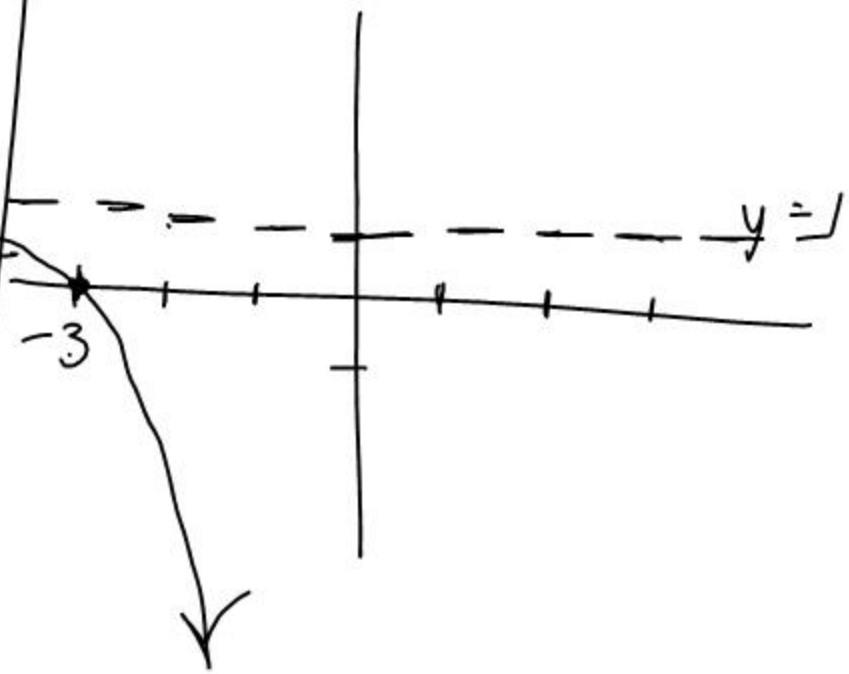
$y = a^x$, $a > 1$

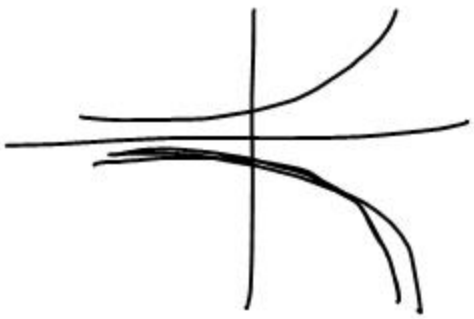
exp. growth



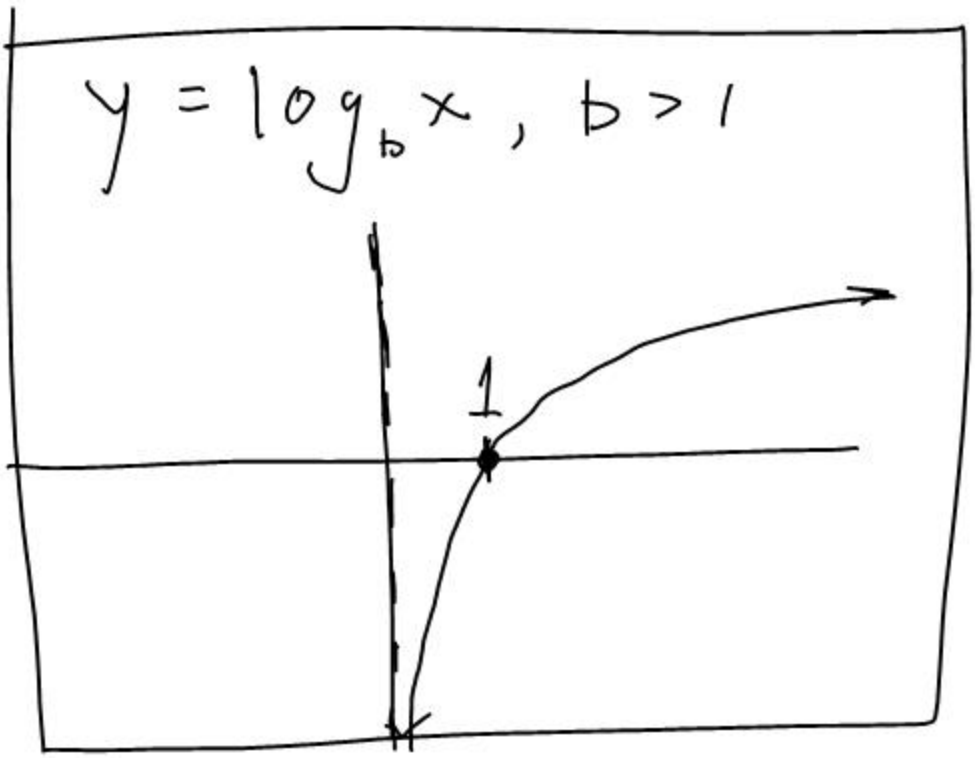
up flips over x-axis left

$y = -2^{x+3} + 1$





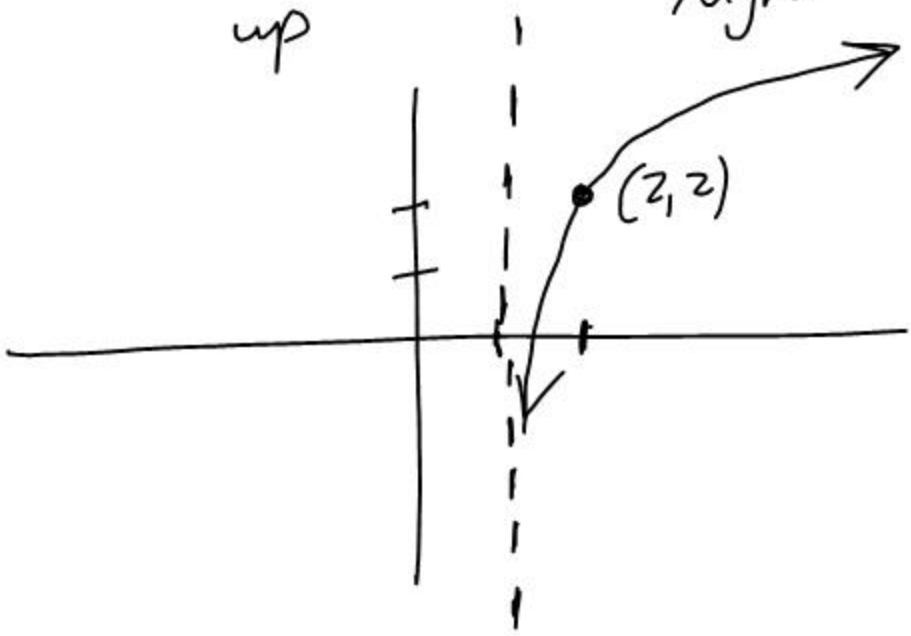
$y = \log_b x, \quad b > 1$



$y = 2 + \log_2(x-1)$

up

right



5 (a) $\log_2 3 + \log_2 y = \log_2 (3y)$

(b) $2\log_2 5 + 3\log_2 x$

$= \log_2 5^2 + \log_2 x^3 = \log_2 (25x^3)$

(c) $\log_3 p - \log_3 q = \log_3 \left(\frac{p}{q}\right)$

(d) $3\log_3 x = 4\log_3 y$

$= \log_3 x^3 - \log_3 y^4 = \log_3 \left(\frac{x^3}{y^4}\right)$

(e) $\ln x - \ln 5 = \ln \left(\frac{x}{5}\right)$

(f) $2\ln 4 - 3\ln y$

$= \ln 4^2 - \ln y^3 = \ln \left(\frac{16}{y^3}\right)$

$\ln x = \log_e x$

#6 (a) $\log_2 4x = \log_2 4 + \log_2 x$

(b) $\log_2 \frac{p}{q} = \log_2 p - \log_2 q$

(d) $\log_3 x^4 - \log_3 y^6$

$= 4\log_3 x - 6\log_3 y$

(c) $\log_3 (4x^3) = \log_3 4 + 3\log_3 x$

(e) $\ln x - \ln 5$

(f) $2 \cdot \ln x - 3 \cdot \ln y$

$\log a + \log b = \log (ab)$

$\log a - \log b = \log \left(\frac{a}{b}\right)$

$a \cdot \log b = \log b^a$

7 (a) $\log_2 5 + \log_2 x = 3$ } condense logs

$\log_2 5x = 3$ ← log form ↔ exp. form

$2^3 = 5x$

$8 = 5x$

$\boxed{\frac{8}{5} = x}$

(b) $\log_3 x + \log_3 (x-2) = 1$

$\log_3 (x^2 - 2x) = 1$ } convert to exponential form

$3^1 = x^2 - 2x$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\boxed{x = 3}$ or ~~$x = -1$~~

(c) $\underbrace{\ln x + \ln 3}_{\ln_e 3x} = 1$

$e^1 = 3x$

$\boxed{\frac{e}{3} = x}$

(d) $2^{5x-1} = 4$

$\ln 2^{5x-1} = \ln 4$

$(5x-1)\ln 2 = \ln 4$

$5x \ln 2 - \ln 2 = \ln 4$

$5x \ln 2 = \ln 4 + \ln 2$

$\boxed{x = \frac{\ln 4 + \ln 2}{5 \ln 2}}$

$= \frac{2\ln 2 + \ln 2}{5 \ln 2} = \frac{3 \cancel{\ln 2}}{5 \cancel{\ln 2}}$

$2^{5x-1} = 2^2$

$5x - 1 = 2$

$5x = 3$

$\boxed{x = \frac{3}{5}}$

(e) $5^{x^2} = 25$

$5^{x^2} = 5^2$

$x^2 = 2$

$x = \pm\sqrt{2}$

(f) $3^x = 5$

$x \cdot \ln 3 = \ln 5$

$x = \frac{\ln 5}{\ln 3}$

(g) $5^{2x} = 12$

$2x \ln 5 = \ln 12$

$x = \frac{\ln 12}{2 \ln 5}$

(h) $e^{2x+3} = 10$

$\ln e^{2x+3} = \ln 10$

$2x + 3 = \ln 10$

$x = \frac{\ln 10 - 3}{2}$

Word Problem Review

① A bacteria culture 1,000,000 bacteria...
5 hours later, there are 1,500,000.

- How many bacteria will there be after 7 hours?
- How long until the pop. doubles?

$$P = 1{,}000{,}000 \cdot b^t$$

$$1{,}500{,}000 = 1{,}000{,}000 \cdot b^5$$

$$1.5 = b^5$$

$$b = 1.5^{1/5} = 1.08447$$

$$\boxed{P = 1{,}000{,}000\,(1.08447)^t}$$

- $P(7) = 1{,}000{,}000\,(1.08447)^7$
  $= 1{,}764{,}119$

$$2{,}000{,}000 = 1{,}000{,}000\,(1.08447)^t$$

$$2 = 1.08447^t$$

$$\ln 2 = t \ln 1.08447 \rightarrow \boxed{t = \frac{\ln 2}{\ln 1.08447}}$$

$$t = 8.5 \text{ hrs}$$

Ex. Carbon-15 has a half life of 2.449 secs. Start with 10 grams.

- How much is left after 6 secs?
- How long will it take to get down to just 1 gram?

$m = 10 \cdot b^t$

$5 = 10 \cdot b^{2.449}$

$\frac{1}{2} = b^{2.449}$

$b = \left(\frac{1}{2}\right)^{\frac{1}{2.449}} = 0.753495$

$\boxed{m = 10(0.753495)^t}$

- $m(6) = 10(0.753495)^6 = \underline{1.83 \text{ gr}}$
- $1 = 10(0.753495)^t$

$0.1 = 0.753495^t$

$\ln 0.1 = t \cdot 0.753495$

$t = \frac{\ln 0.1}{\ln 0.753495}$

$= \underline{8.135 \text{ secs}}$

Optional practice

(1) A town has a pop. of 10000 and grows at 6% each year.

- Find the pop. after 4 years.
- How long will it take to double?

(2) You start with 100 g of a radioactive material. After 10 days, there are 30 g remaining.

- Find the half-life

(3)