Rational Functions

$f(x) = \frac{\text{polynomial}}{\text{polynomial}}$

Ex. $f(x) = \frac{x+1}{x-2}$

x-intercept(s) -1 $\quad 0 = \frac{x+1}{x-2}$

1 y-intercept $-1/2$ $\quad f(x) = \frac{0+1}{0-2} = -1/2$

Vertical asymptote(s) $x=2$ $\quad$ denominator $= 0$

$x - 2 = 0$

$x = 2$

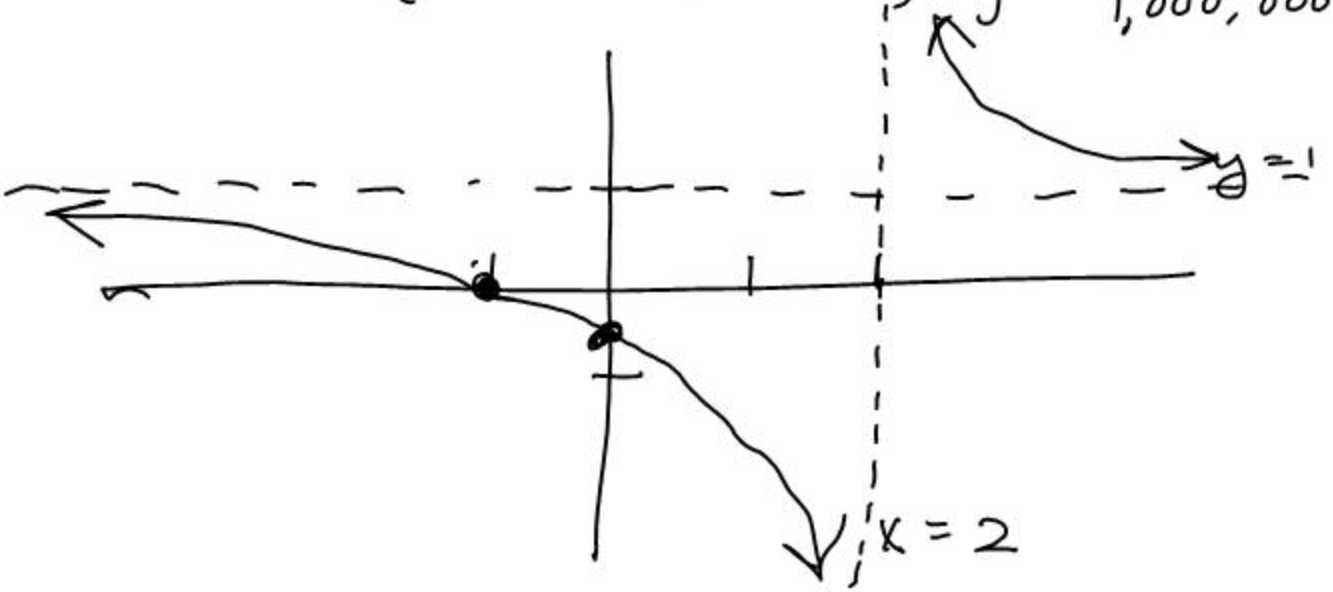
1 non-vertical asymptote $y=1$

(tells us how the function behaves as $x \to \infty$ or $x \to -\infty$)

As $x \to \infty$, (use $x = 1002$) $\quad y = \frac{1002+1}{1002-2} = \frac{1003}{1000}$

$= 1.003$

(use $x = 1{,}000{,}002$) $\quad y = \frac{1{,}000{,}003}{1{,}000{,}000} = 1.000003$



Ex Sketch $f(x) = \dfrac{2x+1}{x-2}$

x-int $-\frac{1}{2}$

y-int $-\frac{1}{2}$

V.A. $x = 2$

N.V.A. $y = 2$

$0 = \dfrac{2x+1}{x-2}$

$2x+1 = 0$

$f(0) = \dfrac{2(0)+1}{0-2}$

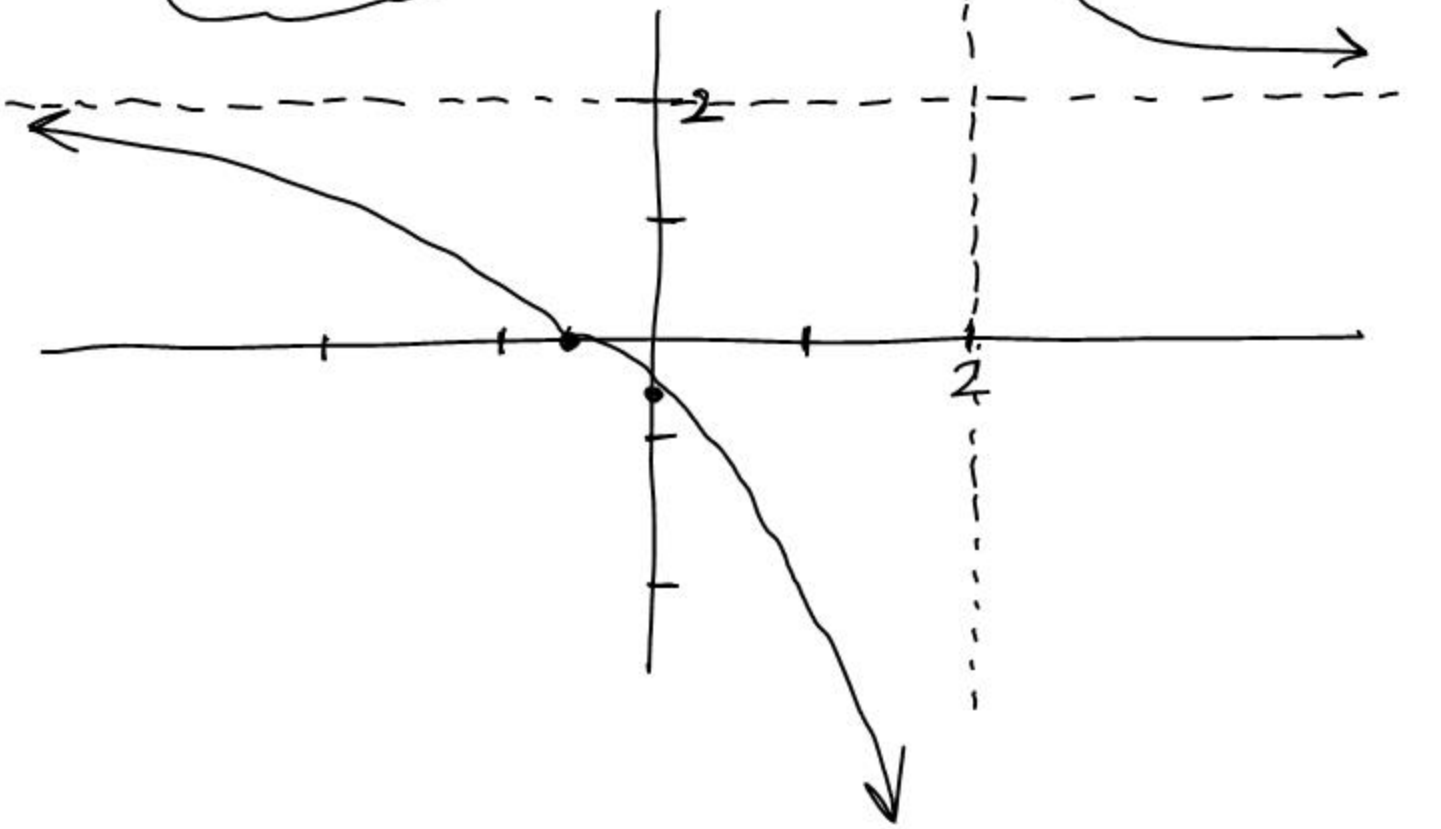
$= -\frac{1}{2}$

$x - 2 = 0$

$x = 2$

$x = 1002$

$\dfrac{2001}{1000} = 2.001$



Ex. $f(x) = \dfrac{x^2 - 4}{x^2 - 9} = \dfrac{(x-2)(x+2)}{(x-3)(x+3)}$

x-int ± 2

y-int $4/9$

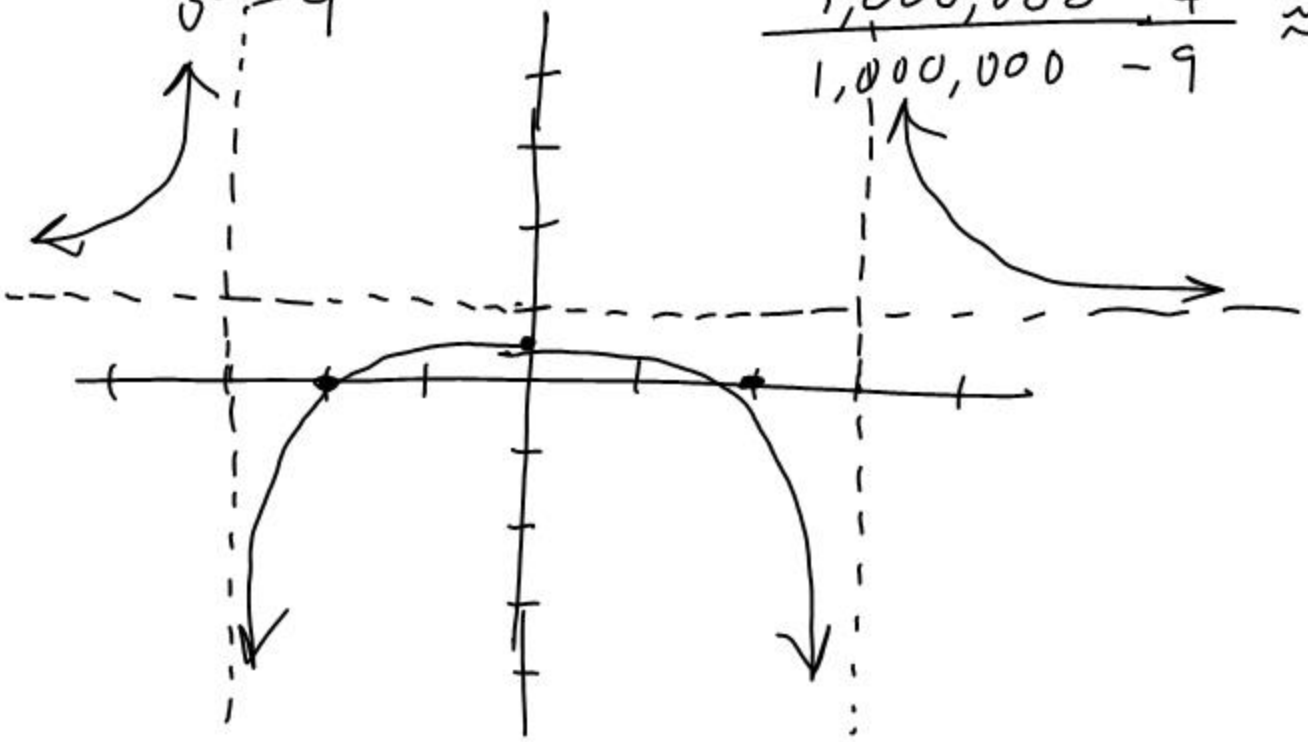
V.A. $x = 3, x = -3$

N.V.A. $y = 1$

$\dfrac{0^2 - 4}{0^2 - 9}$

$x = 1000$

$\dfrac{1{,}000{,}000 - 4}{1{,}000{,}000 - 9} \approx 1$

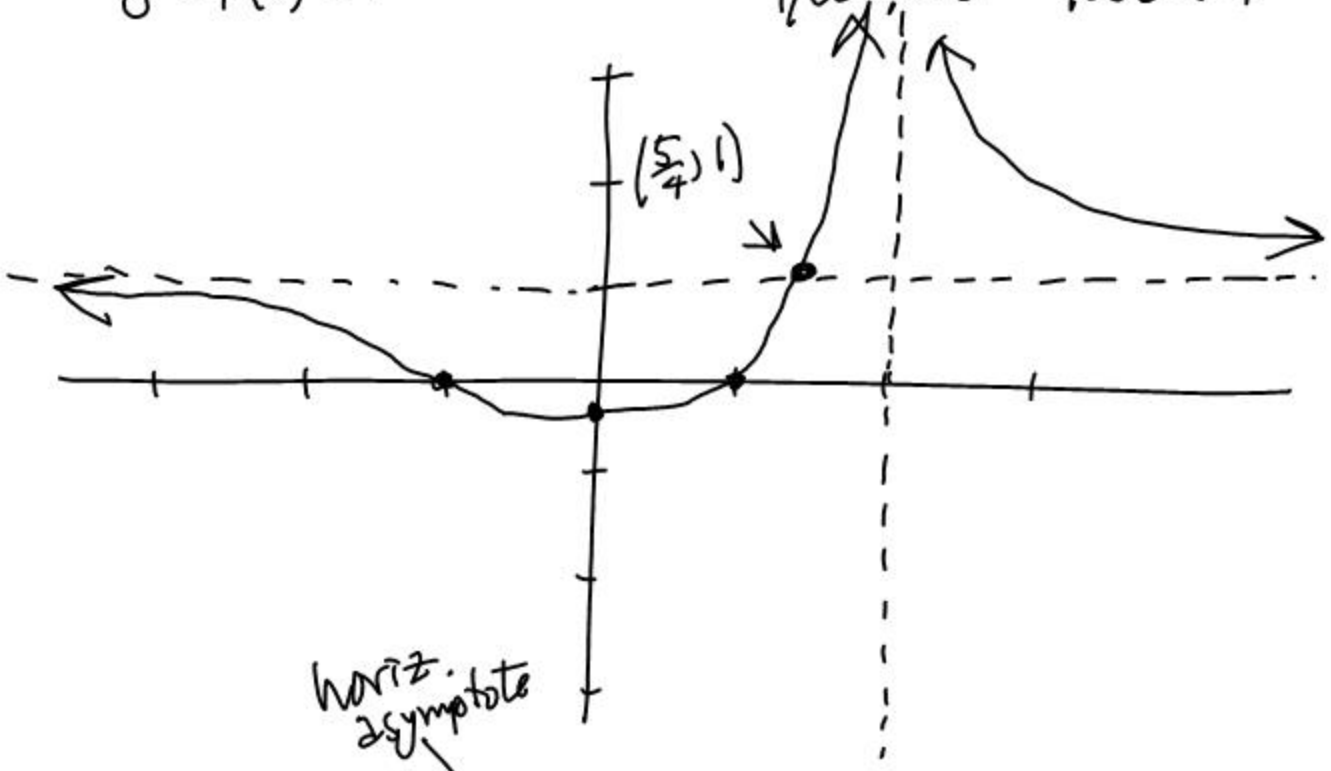


Ex. $f(x) = \dfrac{x^2-1}{x^2-4x+4} = \dfrac{(x-1)(x+1)}{(x-2)(x-2)}$

x-int ± 1 V.A. $x=2$

y-int $-1/4$ H.V.A. $y=1$

$\dfrac{0^2-1}{0^2-4(0)+4}$ $\dfrac{1{,}000{,}000-1}{1{,}000{,}000-4000+4} \approx 1$



horiz. asymptote

$$\frac{x^2-1}{x^2-4x+4} = 1$$

$$x^2-1 = x^2-4x+4$$

$$4x = 5$$

$$x = 5/4 = 1\tfrac{1}{4}$$ ← where the graph crosses the H.V.A.

Ex. $f(x) = \dfrac{x^2 - 4}{x+1} = \dfrac{(x+2)(x-2)}{x+1}$

x-int ± 2

y-int -4

V.A. $x = -1$

N.V.A. $y = x - 1$

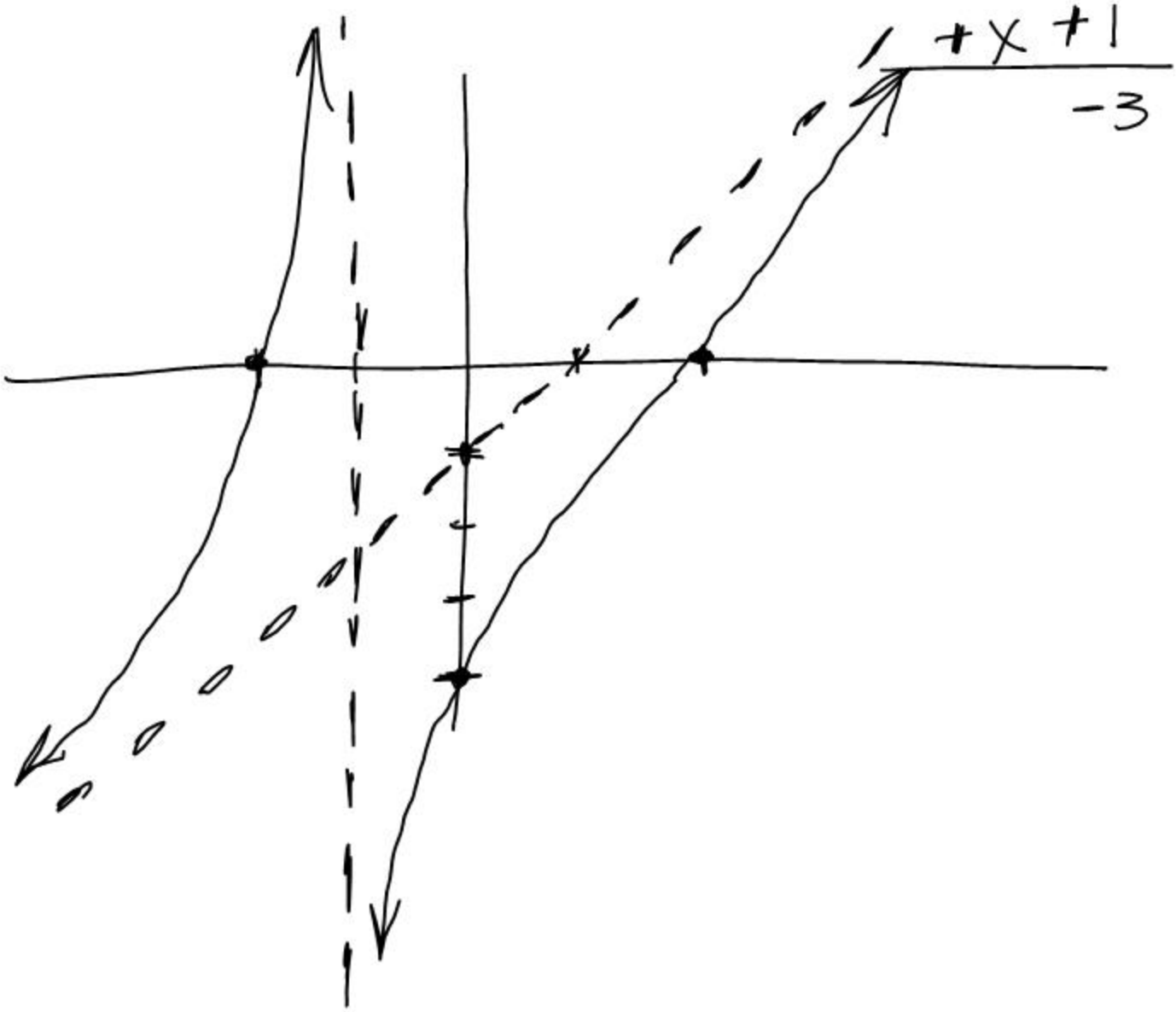
$$x+1 \overline{)\, x^2 - 4} \quad x - 1 - \frac{3}{x+1}$$

$$-x^2 - x$$

$$-x - 4$$

$$+x + 1$$

$$-3$$



HW Sketch

① $y = \dfrac{3x+1}{2x-1}$

② $y = \dfrac{x^2-1}{x^2-4}$

③ $y = \dfrac{x^2}{(x+1)^2}$

④ $y = \dfrac{x^2-9}{x-1}$