① $\dfrac{3+\sqrt{-50}}{2} = \dfrac{3+5\sqrt{2}\,i}{2}$ $\quad \sqrt{-1\cdot 25\cdot 2}$

$5\sqrt{2}\,i$

$= \dfrac{3}{2} + \dfrac{5\sqrt{2}}{2}i$

② $(5+4i)(5+4i)$

$= 25 + 20i + 20i \cancel{+16i^2}^{\,-16} = 9 + 40i$

$\sqrt{9+40i} = 5+4i$

③ $x^2 - 4x + 8 = 0$

$x = \dfrac{4 \pm \sqrt{16-32}}{2} = \dfrac{4 \pm \sqrt{-16}}{2}$

$= \dfrac{4 \pm 4i}{2} = \underline{\underline{2 \pm 2i}}$

④ $y = x^2 + 12x - 9$

$= (x^2 + 12x \underline{+36}) - 9 \underline{-36}$

$y = (x \underline{+ 6})^2 - 45 \quad V(-6, -45)$

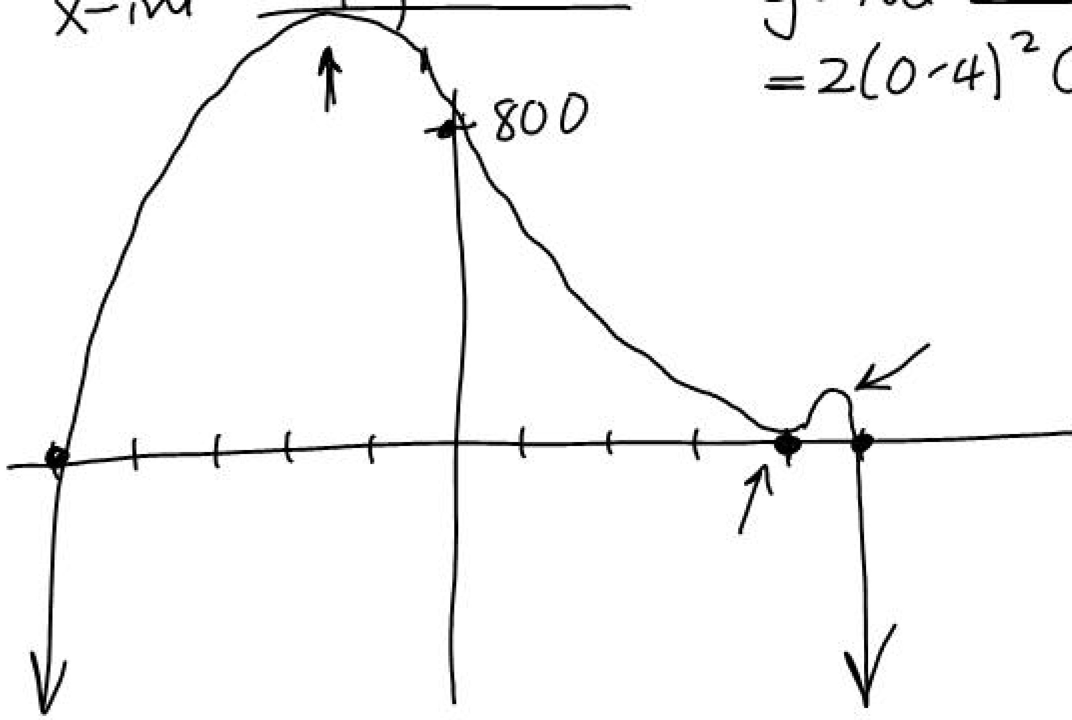
56 $y = -2(x-4)^2(x^2-25)$

$y = -2(\underline{x-4})^2(\underline{x-5})(\underline{x+5})$

x-int $\underline{4, \pm 5}$

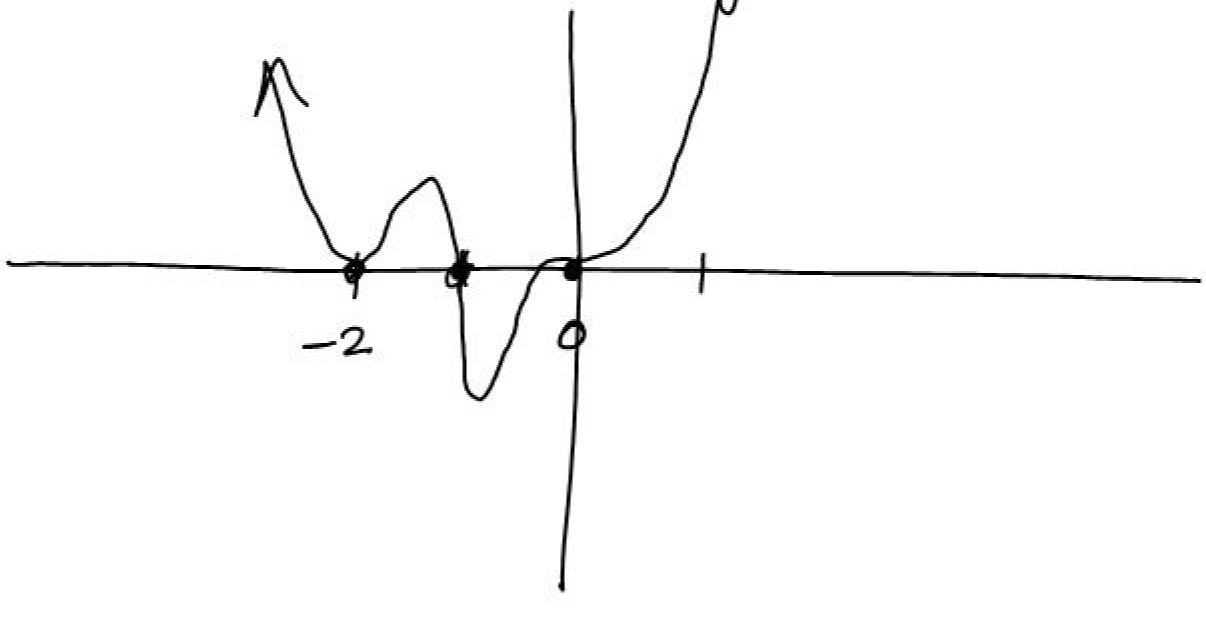
y-int ___

$= 2(0-4)^2(0^2-25) = +800$



#58 $y = x^3(x+2)^2(x+1)$

x-int $\underline{0, -2, -1}$ y-int $\underline{0}$

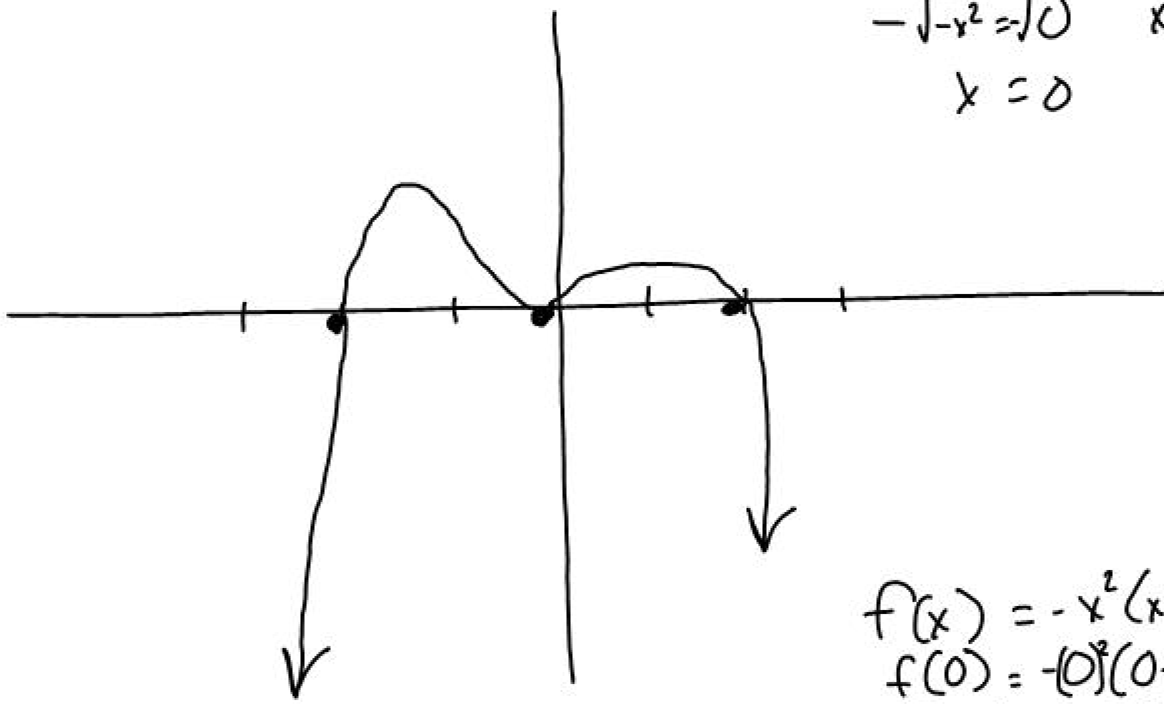


#60 $f(x) = -x^2(x+2)(x-2)$

x-int $\underline{0, -2, 2}$ y-int $\underline{0}$

$-\sqrt{-x^2} = \sqrt{0}$ $x+2=0$ $x-2=0$

$x = 0$ $x=-2$ $x=2$



$f(x) = -x^2(x+2)(x-2)$

$f(0) = -(0)^2(0+2)(0-2)$

$f(0) = 0(2)(-2) = 0$

#41. $y = x^3 + 2x^2 - x - 2$

possible zeros

$\pm 1, \pm 2$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -1 & -2 \\ & \downarrow & 1 & 3 & 2 \\ \hline & 1 & 3 & 2 & \boxed{0} \end{array} = f(1)$$

$y = (x-1)(x^2+3x+2)$

$y = (x-1)(x+1)(x+2)$

Solve: $x^4 - 6x^2 - 8x + 24 = 0$

zeros

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -6 & -8 & 24 \\ & & 2 & 4 & -4 & -24 \\ \hline & 1 & 2 & -2 & -12 & \boxed{0} = f(2) \end{array}$$

$$(x-2)(x^3 + 2x^2 - 2x - 12) = 0$$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -2 & -12 \\ & & 2 & 8 & 12 \\ \hline & 1 & 4 & 6 & \boxed{0} \end{array}$$

$$(x-2)(x-2)(x^2 + 4x + 6) = 0$$

$\boxed{x = 2}$

double root

$$x = \frac{-4 \pm \sqrt{16 - 24}}{2}$$

$\sqrt{-1 \cdot 4 \cdot 2}$

$$= \frac{-4 \pm \sqrt{-8}}{2}$$

$$= \frac{-4 \pm 2\sqrt{2}\, i}{2}$$

$$= \frac{\cancel{2}(-2 \pm \sqrt{2}\, i)}{\cancel{2}} = -2 \pm \sqrt{2}\, i$$

Descartes' Rule of Signs

Predicting the number of real roots (zeros)

Ex. $+4x^4+3x^3+x^2+5x+1=0$ no sign changes

How many positive roots : $\underline{\ 0\ }$

Ex. $x^3-4x^2+5x-6=0$

3 sign changes

3 positive roots

or 1 positive root

replace x with $-x$

$$(-x)^3-4(-x)^2+5(-x)-6=0$$

$$-x^3-4x^2-5x-6=0$$

no sign changes

$\Rightarrow$ no negative roots

$\boxed{\text{3 positives}}$ or $\boxed{\text{1 positive and 2 imaginary roots}}$

Ex Predict the type of roots:

$$2x^4 - 3x^3 + 4x^2 + x + 5 = 0$$

pos. roots: 2 or 0

$$2x^4 + 3x^3 + 4x^2 - x + 5 = 0$$

neg. roots: 2 or 0

Solve: $x^3 + 4x^2 - 3x - 6 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 4 & -3 & -6 \\ & & 1 & 5 & 2 \\ \hline & 1 & 5 & 2 & \boxed{-4} = f(1) \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 4 & -3 & -6 \\ & & -1 & -3 & 6 \\ \hline & 1 & 3 & -6 & \boxed{0} \end{array}$$

$$(x+1)(x^2+3x-6) = 0$$

$$\boxed{x = -1} \qquad x = \frac{-3 \pm \sqrt{9 - -24}}{2} = \boxed{\frac{-3 \pm \sqrt{33}}{2}}$$

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35 – 38 Descartes' Rule