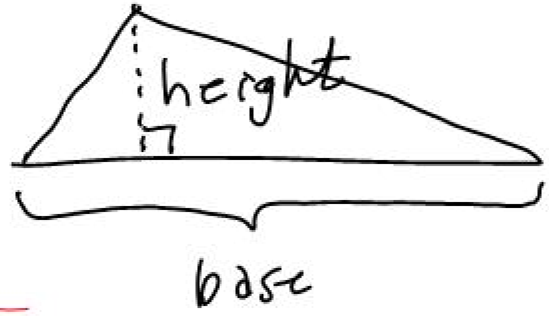
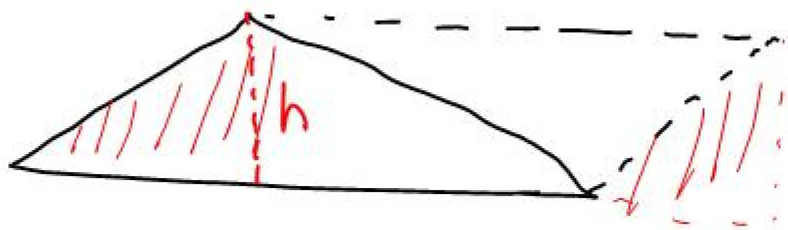
The Area of a Triangle

1st formula

$$\boxed{A = \frac{1}{2} b h}$$

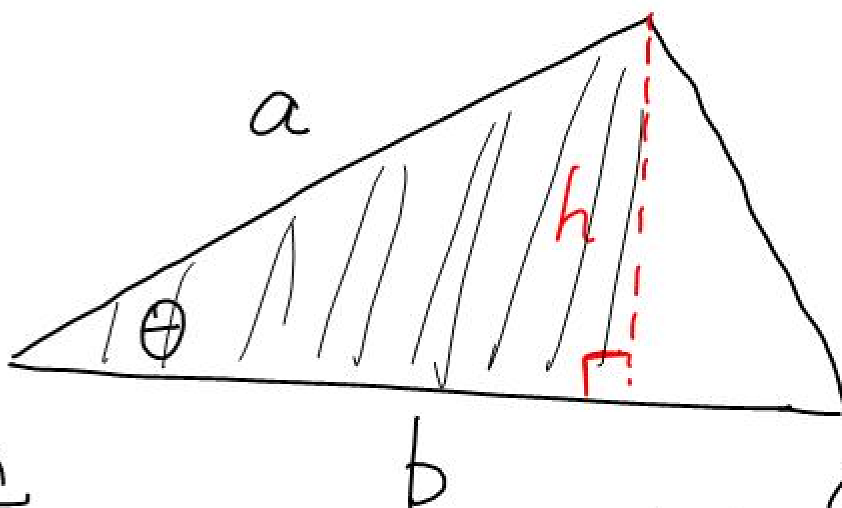


parallelogram area is $b \cdot h$



2nd formula (SAS formula)

We know the values of a, b, and θ.



$$\sin\theta = \frac{h}{a}$$

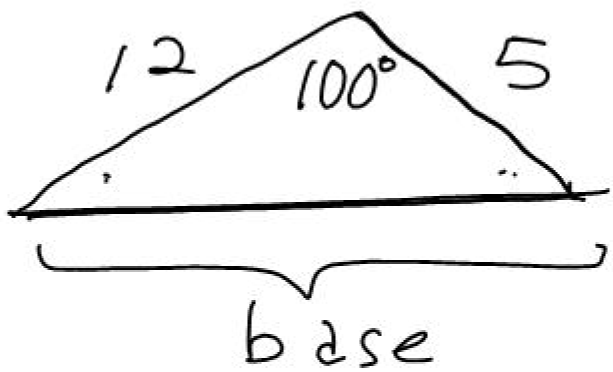
$$\boxed{h = a \cdot \sin\theta}$$

$$A = \frac{1}{2} b \cdot \overset{a \cdot \sin\theta}{\cancel{h}}$$

$$\boxed{A = \frac{1}{2} ab\sin\theta}$$

SAS area formula: the area of a triangle is $\frac{1}{2}$ the product of two sides times the sine of the angle between the two sides.

EX. (easy)

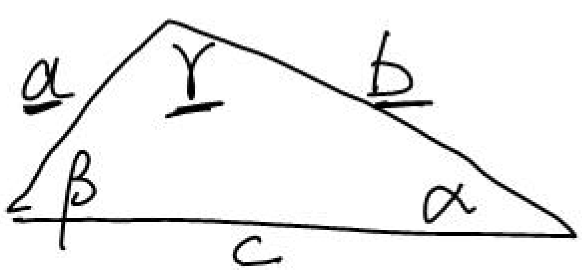


Find (a) area, (b) base, and (c) height.

(a) $\text{Area} = \frac{1}{2} \cdot \underbrace{(12)(5)}_{\text{product of 2 sides}} \sin 100^\circ = 29.5$

angle between the 2 sides

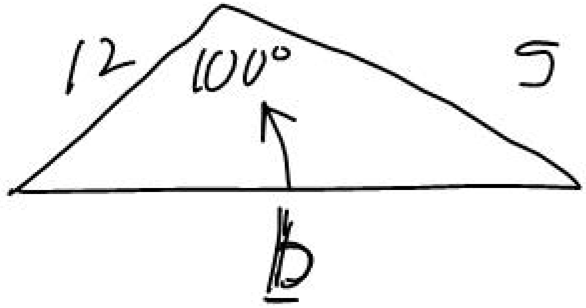
(b) <u>The Cosine Rule</u> (SAS or SSS)



$c^2 = a^2 + b^2 - 2ab\cos\gamma$

opposite each other

Finding the base



$$b^2 = \boxed{12}^2 + \boxed{5}^2 - 2 \cdot \boxed{12} \cdot \boxed{5} \cos \boxed{100^\circ}$$

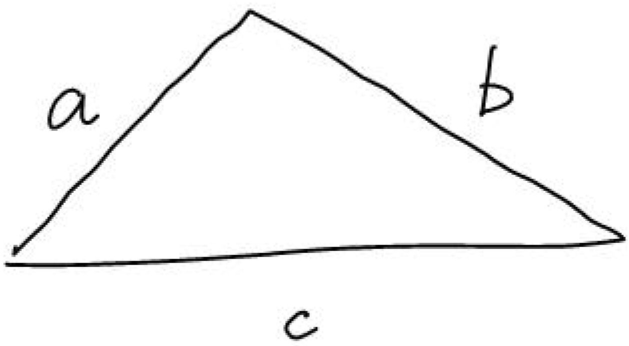
(c) Find the height.

$$A = \frac{1}{2} b \cdot h \Rightarrow h = \frac{2A}{b}$$

$$h = \frac{2(29.544)}{13.778} = \underline{\underline{4.29}}$$

3rd formula (SSS formula)

Heron's Formula

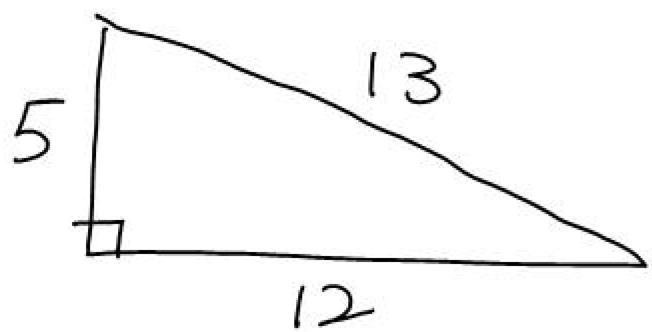


Semiperimeter

$$s = \frac{1}{2}(a+b+c)$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Ex.



$$s = \frac{1}{2}(5+12+13)$$

$$s = 15$$

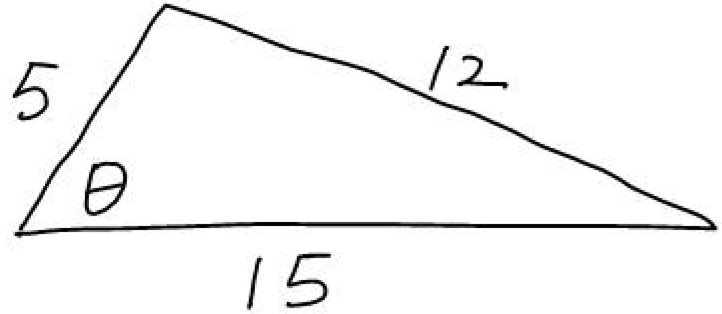
$$\text{Area} = \sqrt{15(15-5)(15-12)(15-13)}$$

$$= \sqrt{15 \cdot 10 \cdot 3 \cdot 2}$$

$$= \sqrt{900}$$

$$= 30$$

Ex.



(a) Find the area (b) Find θ.

(a) $s = \frac{1}{2}(5+12+15) = 16$

$$\text{Area} = \sqrt{16(16-5)(16-15)(16-12)}$$

$$= \sqrt{16 \cdot 11 \cdot 1 \cdot 4}$$

$$= 4 \cdot 2 \cdot \sqrt{11} = 8\sqrt{11}$$

(b) <u>Find θ</u> <u>Method 1</u>

$$8\sqrt{11}\ \cancel{*} = \frac{1}{2}(5)(15)\sin\theta$$

$$\frac{2 \cdot 8\sqrt{11}}{75} = \sin\theta$$

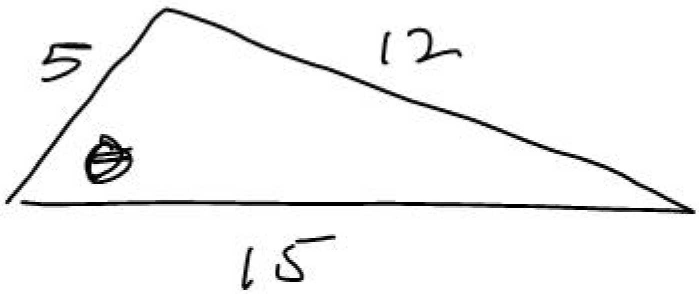
$$0.707546 = \sin\theta$$

$\theta = 45.0°$

take the inverse sine

Find θ. Method 2

Use the Cosine Rule. (SSS)



$$12^2 = 5^2 + 15^2 - 2 \cdot 5 \cdot 15 \cdot \cos\theta$$

$$144 = 250 - 150\cos\theta$$

$$-250 \qquad -250$$

$$\frac{-106}{-150} = \frac{-150\cos\theta}{-150}$$

$$0.706667 = \cos\theta$$

take the inverse cosine

$$\theta = 45.0^\circ$$

The Cosine Rule

SAS : $c^2 = a^2 + b^2 - 2ab\cos C$

SSS : $\cos\theta = \dfrac{a^2+b^2-c^2}{2ab}$