

Exam Review

-1 +5 -2

① Solve $x^3 - 5x - 2 = 0$

zero → -2

factor ↓

$$\begin{array}{r|rrrr} & 1 & 0 & -5 & -2 \\ & & -2 & 4 & 2 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$$(x+2)(x^2 - 2x - 1) = 0$$

quadratic ← factor

$$b^2 - 4ac = 4 + 4 = 8 \quad \leftarrow \text{quad. formula}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = -2$$

$$x = 1 \pm \sqrt{2}$$

$$\begin{aligned} & \sqrt{8} \\ &= \sqrt{4 \cdot 2} \\ & \quad \downarrow \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

② Solve: $x^3 + x^2 + 4x + 4 = 0$

zero \rightarrow -1 | 1 | 1 | 4 | 4
factor \downarrow | | -1 | 0 | -4

| 1 | 0 | 4 | 0

$$(x+1)(x^2+4) = 0$$

$$\downarrow$$
$$x^2 + 4 = 0$$

$$x^2 = -4$$

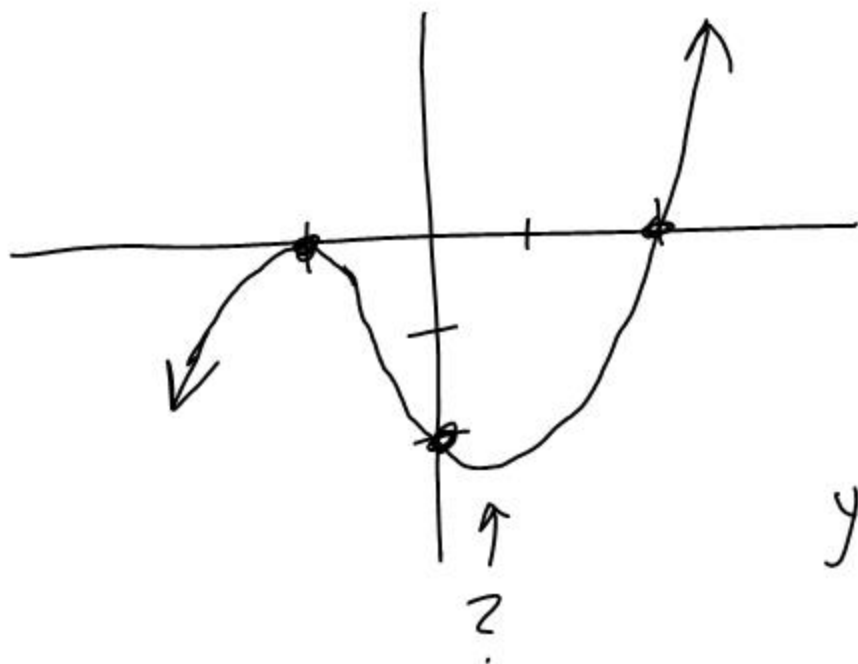
$$x = \pm \sqrt{-4}$$

$$x = \pm 2i$$

$$x = -1$$

$$\sqrt{-4}$$
$$\sqrt{-1 \cdot 4}$$
$$i \cdot 2$$

(3) Sketch $y = (x+1)^2(x-2)$

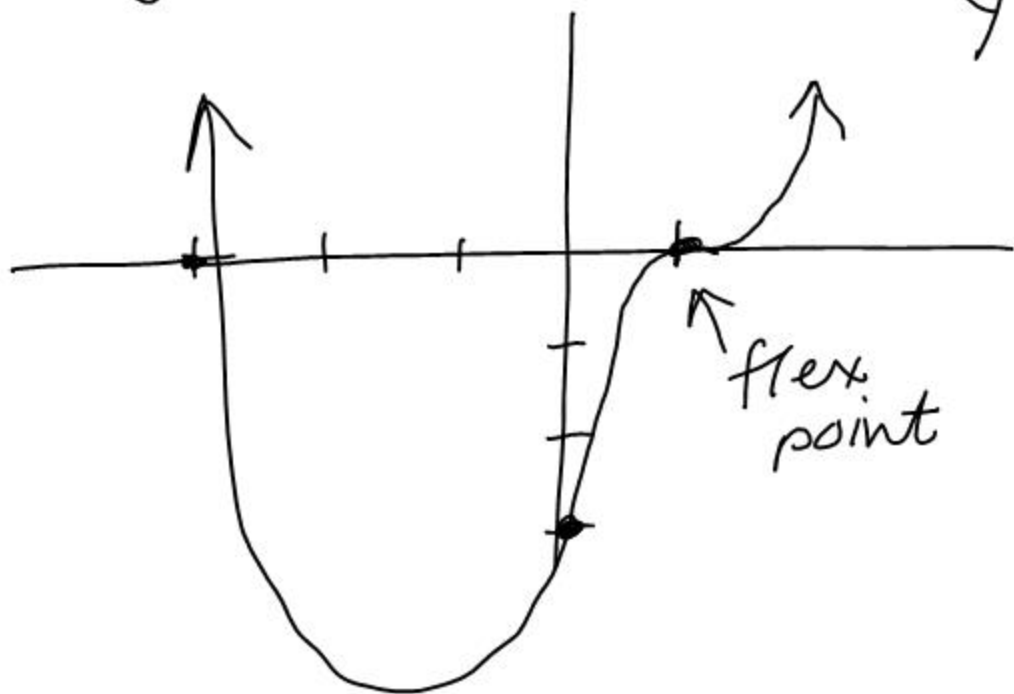


x-intercepts
-1, 2

y-intercept

$$y = (0+1)^2(0-2) = -2$$

(4) $y = (x-1)^3(x+3)$



$$y = (0-1)^3(0+3) = (-1)(3) = -3$$

5 Sketch

$$y = \frac{x+2}{x-3}$$

$$\frac{0+2}{0-3}$$

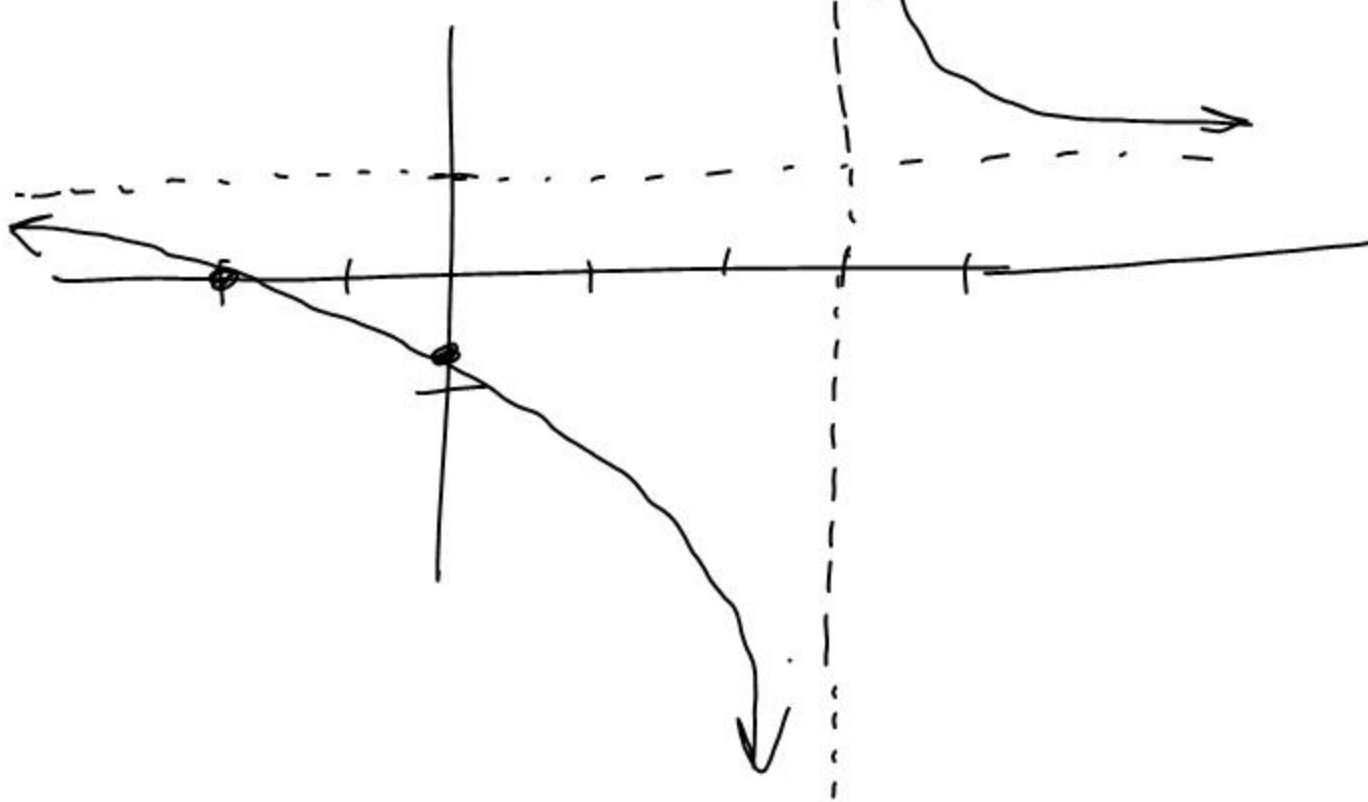
rational
functions

x-intercept $\underline{-2}$

y-intercept $\underline{-\frac{2}{3}}$

vertical asymptote $\underline{x=3}$

horizontal asymptote $\underline{y=1}$



⑥ Sketch

$$y = \frac{x^2 - 4x + 4}{x^2 + 4x + 3} = \frac{(x-2)^2}{x^2 + 4x + 3}$$

x-int $\frac{2}{3}$

y-int $\frac{4}{3}$

VA $x = -3, x = -1$

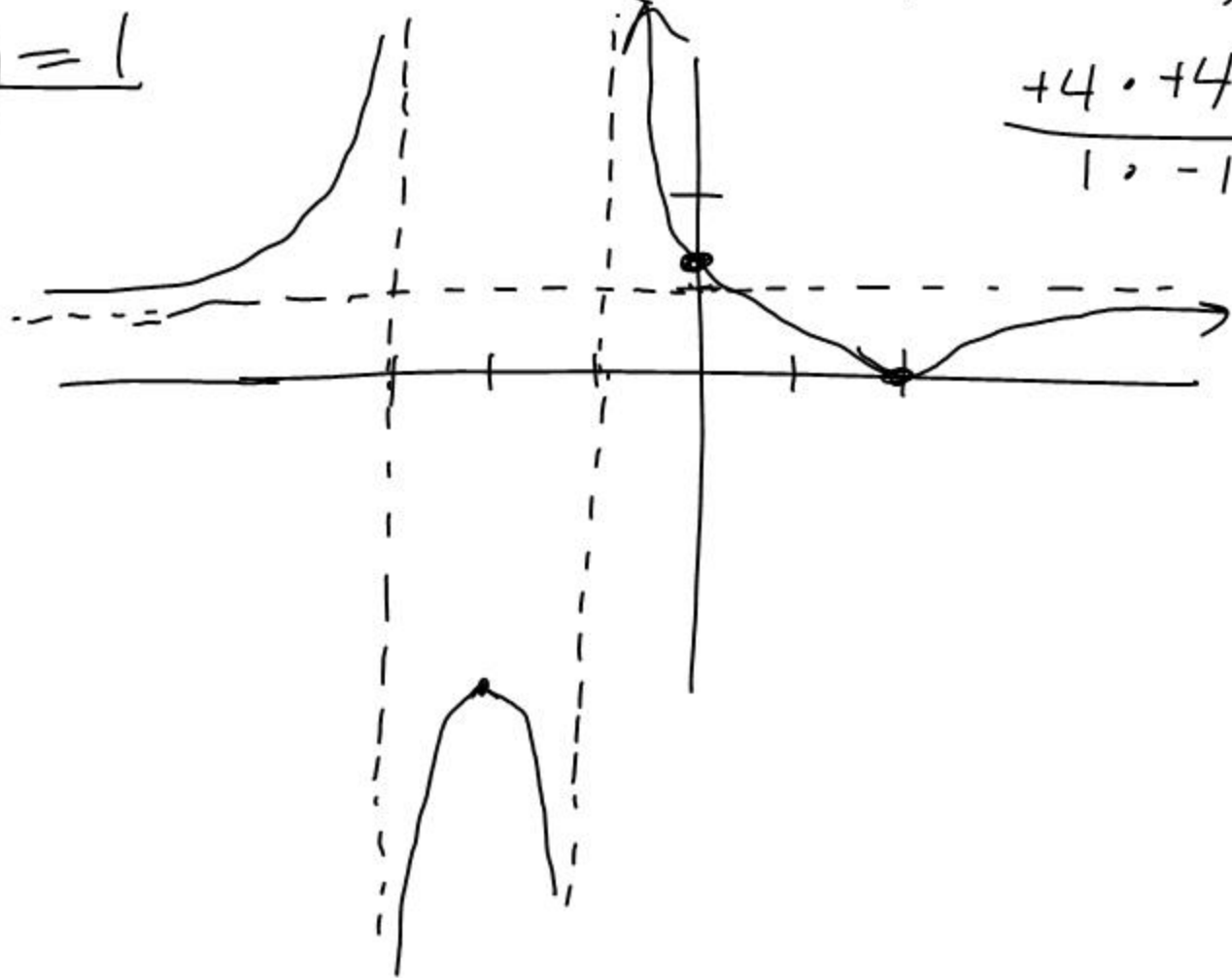
HA $y = 1$

$$y = \frac{(x-2)^2}{(x+3)(x+1)}$$

$$(x-2)(x-2)$$

$$(x+3)(x+1)$$

$$\frac{+4 \cdot +4}{1 \cdot -1}$$



$$\textcircled{1} (a) \log_5 \left(\frac{1}{125} \right) = \boxed{-3} \quad \left(\text{---} \right)$$

$$5^{\boxed{-3}} = \frac{1}{125} = \frac{1}{5^3} = 5^{-3}$$

$$(b) \log_{10} \sqrt{1000} = \frac{3}{2}$$

$$10^{\boxed{3/2}} = \sqrt{1000} = \sqrt{10^3} \\ = (10^3)^{1/2} = 10^{3/2}$$

$$(c) \ln e^2 = 2$$

$$\ln x = \log_e x$$

$$e^{\boxed{2}} = e^2$$

$$e) \ln 1 = 0$$

$$(d) \ln \frac{1}{e} = -1$$

$$e^{\boxed{0}} = 1$$

$$e^{\boxed{-1}} = \frac{1}{e}$$

⑧ Solve:

$$2^{x+3} = 9$$

exponential equation
(variable in the exponent)

$$\ln 2^{x+3} = \ln 9$$

log
rule

$$\ln a^b = b \cdot \ln a$$

$$(x+3) \ln 2 = \ln 9$$

$$\begin{aligned} x \cdot \ln 2 + 3 \cdot \ln 2 &= \ln 9 \\ -3 \cdot \ln 2 &\quad -3 \ln 2 \end{aligned}$$

$$\frac{x \cdot \cancel{\ln 2}}{\cancel{\ln 2}} = \frac{\ln 9 - 3 \ln 2}{\ln 2}$$

$$x = \frac{\ln 9 - 3 \ln 2}{\ln 2}$$

9) Solve: $8^x = 32^{x-1}$

$$(a^m)^n = a^{mn}$$

$$(2^3)^x = (2^5)^{x-1}$$

$$2^{3x} = 2^{5x-5}$$

$$3x = 5x - 5$$

$$5 = 2x$$

$$\frac{5}{2} = x$$

(10) A town has 8000 people and is growing at 2.1% per year.

(a) Find the pop. in 10 years.

(b) Find how long it will take the pop. to double.

$$P = a \cdot b^t \text{ where } b = 1 + r$$

growth
rate
↓

$$P = 8000 \cdot (1 + 0.021)^t$$

$$P = 8000(1.021)^t$$

$$(a) P(10) = 8000(1.021)^{10} = 9848$$

$$(b) 16000 = 8000(1.021)^t$$

$$(b) 16000 = 8000 (1.021)^t$$

$$2 = 1.021^t$$

$$\frac{\ln 2}{\ln 1.021} = \frac{t \cdot \ln 1.021}{\ln 1.021}$$

$$t = \frac{\ln 2}{\ln 1.021} \approx 33.4 \text{ yrs}$$

Approximate doubling time:
The rule of 70

$$\frac{70}{2.1} = 33.3 \text{ yrs}$$

⑥ Trig

$$(a) \sin \frac{11\pi}{4} \\ = \frac{1}{\sqrt{2}}$$

