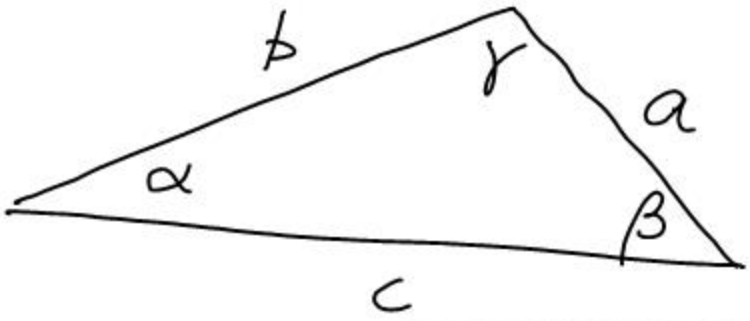
The Sine Rule



$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

(1) $\dfrac{\sin 40^\circ}{15} = \dfrac{\sin 95^\circ}{b}$

$b \cdot \sin 40^\circ = 15 \cdot \sin 95^\circ$

$$b = \frac{15 \cdot \sin 95^\circ}{\sin 40^\circ}$$

$b = 23.2$

(2) $\frac{\sin 105^\circ}{42} = \frac{\sin 60^\circ}{c}$

$c \cdot \cancel{\sin 105^\circ} = \frac{42 \cdot \sin 60^\circ}{\sin 105^\circ}$

$\boxed{c = 37.7}$

(3) $\alpha = 26^\circ \qquad a = \_\_\_$

$\beta =$

$\gamma = 55^\circ \qquad c = 10$

$\frac{\sin 55^\circ}{10} = \frac{\sin 26^\circ}{a}$

$a = \frac{10 \cdot \sin 26^\circ}{\sin 55^\circ} = 5.35$

ASA

$\alpha = 180° - (115° + 48°) = 17°$

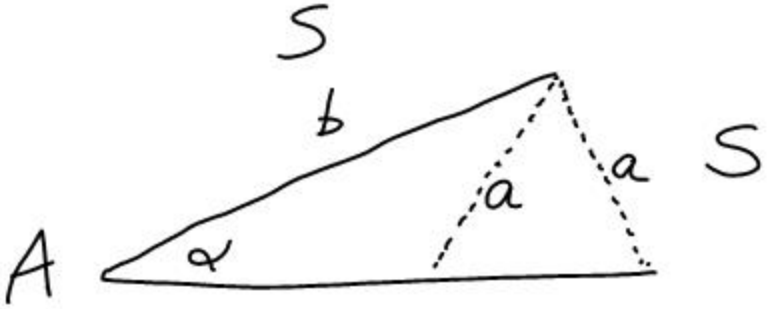
⑥ $\frac{\sin 115°}{b} = \frac{\sin 17°}{9}$

$b \cdot \sin 17° = \frac{9 \cdot \sin 115°}{\sin 17°}$

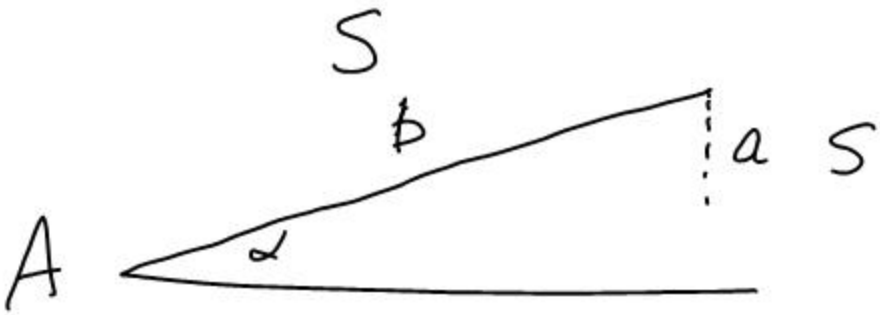
$\boxed{b = 27.9}$

SSA

2 triangles



no triangle



1 triangle

The Ambiguous case of the Sine Rule (SSA)

Ex. $a = 54$, $c = 84$, $\alpha = 31°$

Find b.

$$\frac{\sin 31°}{54} = \frac{\sin \gamma}{84}$$

$$54 \cdot \sin \gamma = 84 \cdot \sin 31°$$

$$\sin \gamma = \frac{84 \cdot \sin 31°}{54} = 0.801170$$

$$\gamma_1 = \sin^{-1}(0.801170) = 53.2° \checkmark$$

or $\gamma_2 = 180 - 53.2° = 126.8°$ ✓

$$\begin{array}{r} 126.8° \\ +\ 31.0 \\ \hline 157.8 \end{array}$$

↑ less than 180°

We have 2 triangles

Triangle 1

$a = 54$	$\alpha = 31°$
$b = 104$	$\beta = 95.8°$
$c = 84$	$\gamma = 53.2°$

$\beta = 180° - (31° + 53.2°)$

$\frac{\sin 95.8°}{b} = \frac{\sin 31°}{54}$

$b = \frac{54 \cdot \sin 95.8°}{\sin 31°}$

Triangle 2

$a = 54$	$\alpha = 31°$
$b = 39.6$	$\beta = 22.2°$
$c = 84$	$\gamma = 126.8°$

$\beta = 180° - (31° + 126.8°)$

$\beta = 22.2°$

$\frac{\sin 22.2°}{b} = \frac{\sin 31°}{54}$

$b = \frac{54 \cdot \sin 22.2°}{\sin 31°}$

Ex. $a = 85$, $c = 84$, $\alpha = 31°$

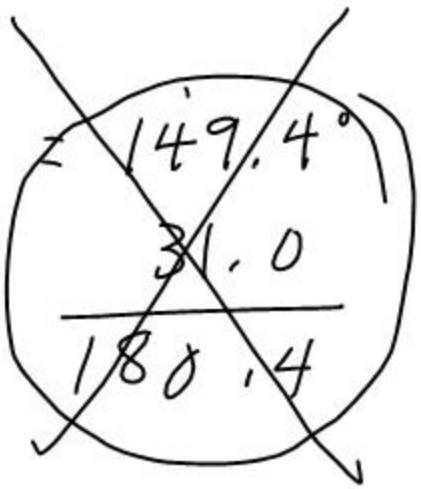
Find b.

$$\frac{\sin 31°}{85} = \frac{\sin \gamma}{84}$$

$$\sin \gamma = \frac{84 \cdot \sin 31°}{85} = 0.508979$$

$\gamma_1 = 30.6°$ ✓

$\gamma_2 = 180° - 30.6° = 149.4°$



only 1 triangle

$a = 85$	$\alpha = 31°$
$b = 145$	$\beta = 118.4°$
$c = 84$	$\gamma = 30.6°$

$$\frac{\sin 31°}{85} = \frac{\sin 118.4°}{b}$$

$$b = \frac{85 \cdot \sin 118.4°}{\sin 31°}$$

Ex. $a = 40$, $c = 84$, $\alpha = 31^\circ$

Find b.

$$\frac{\sin 31^\circ}{40} = \frac{\sin \gamma}{84}$$

$$\sin \gamma = \frac{84 \cdot \sin 31^\circ}{40}$$

$\sin \gamma = 1.08158$ no such angle

No such triangle

#12 $\beta = 80^\circ \quad b = 10, \; c = 11$

Find γ.

$$\frac{\sin 80^\circ}{10} = \frac{\sin \gamma}{11}$$

$$\sin \gamma = \frac{11 \sin 80^\circ}{10} = 1.083$$

No such triangle

#13 $\beta = 80°$ $b = 10$ $c = 9$ Find γ

$$\frac{\sin 80°}{10} = \frac{\sin \gamma}{9}$$

$$\sin \gamma = \frac{9 \cdot \sin 80°}{10} = 0.886327$$

$\gamma_1 = 62.4$

$\gamma_2 = 180 - 62.4 = 117.6°$