#8

$m(t) = 100 \cdot b^t$

m = mass in grams

t = time in days

plug in $(t, m) = (330, 50)$

↑ half of the initial value

$$\frac{50}{100} = \frac{100 \cdot b^{330}}{100}$$

$(a^m)^n = a^{mn}$

$$(1/2)^{\frac{1}{330}} = \left(b^{330}\right)^{\frac{1}{330}}$$

$y = a \cdot b^x$

$0.99790 = b$

$$\boxed{m(t) = 100\,(0.99790)^t}$$

$1 + r = 0.99790$

$r = -0.00209$

(b) growth rate: -0.209% per day

(c) $m(365) = 100(0.99790)^{365}$

$= 46.4\ g$

(d) $\frac{m}{100} = \frac{100(0.99790)^t}{100}$

$\frac{70}{100} = \frac{100(0.99790)^t}{100}$

$0.7 = 0.99790^t$

$\ln 0.7 = t \cdot \ln 0.99790$

$\frac{\ln 0.7}{\ln 0.99790} = \frac{t \cdot \ln 0.99790}{\ln 0.99790}$

$\frac{\ln 0.7}{\ln 0.99790} = t \approx 169.7$ days

[170. days]

More on Exponential Decay

Need to calculate ½-life

Ex. If we start with 72g of precalcium, there are 50g remaining after 43 days.

(a) Find the ½ life.

(b) How long it will take to decay to 10g?

$m(t) = 72 \cdot b^t$ mass in g, time in days

plug in $(t, m) = (43, 50)$

$\frac{50}{72} = \frac{72 \cdot b^{43}}{72}$

$(25/36)^{\frac{1}{43}} = (b^{43})^{\frac{1}{43}}$

$b = 0.9915$

m_2 $t_{\frac{1}{2}}$

$$\boxed{m(t) = 72\,(0.99156)^t}$$

$$36 = 72\,(0.99156)^t$$

t ↑ ½-life

$$\tfrac{1}{2} = 0.99156^t$$

$$\frac{\ln \frac{1}{2}}{\ln 0.99156} = \frac{t \cdot \ln 0.99156}{\ln 0.99156}$$

$$\boxed{\text{half life} = t = 81.8 \text{ days}}$$

(b) $$10 = 72\,(0.99156)^t$$

$$\tfrac{10}{72} = 0.99156^t$$

$$\ln \tfrac{10}{72} = t \cdot \ln 0.99156$$

$$t = \frac{\ln \frac{10}{72}}{\ln 0.99156} \approx \underline{233} \text{ days}$$

HW quiz 11-8-18

$$m(t) = 44(0.98321)^t$$

mass in grams, time in hours

(a) How much remains after 10 hours?

(b) How long before only 5 g remain?

A few more exponential equations

Ex. $e^{2x} - 5e^x + 4 = 0$

$e^{2x} = (e^x)^2$ Let $e^x = y$

$$y^2 - 5y + 4 = 0$$

$$(y-4)(y-1) = 0$$

$y = 4$ or $y = 1$

$e^x = 4$ or $e^x = 1$

$\ln e^x = \ln 4$ $\quad x \cdot \ln e = \ln 1$

$\boxed{x = \ln 4}$ or $\boxed{x = 0}$

check

$2 \cdot \ln 4$

$e^{2\ln 4} - 5e^{\ln 4} + 4$

$e^{\ln 16} - 5e^{\ln 4} + 4$

$16 - 5 \cdot 4 + 4 = 0$ ✓

check $x = 0$

$e^{2 \cdot 0} - 5e^0 + 4$

$1 - 5 + 4 = 0$ ✓

Ex. $e^{2x} - e^x - 6 = 0$

Let $y = e^x$ $\quad e^{2x} = (e^x)^2 = y^2$

$$y^2 - y - 6 = 0$$

$$(y - 3)(y + 2) = 0$$

$y = 3$ or $y = -2$

$e^x = 3$ or $e^x = -2$ no solution

$\ln e^x = \ln 3$

$\boxed{x = \ln 3}$

$$y = \frac{1 \pm \sqrt{1 + 24}}{2} = \frac{1 \pm 5}{2}$$

$= 3$ or -2

HW Logarithms #7ij

Word problems #12-14

Monday: Test Review

Wed: some new material

Fri: TEST