4, 9, 16, 25, 36, 49, ...

Simplify $\boxed{\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}}$

$$\sqrt{50} = \sqrt{25 \cdot 2}$$
$$= \sqrt{25} \cdot \sqrt{2}$$
$$= 5\sqrt{2}$$

$$\sqrt{80} = \sqrt{16 \cdot 5}$$
$$= \sqrt{16} \cdot \sqrt{5}$$
$$= 4\sqrt{5}$$

80

$4 \cdot 20$

$\underbrace{4 \cdot 4} \cdot 5$

#5 $\boxed{P(t) = a \cdot b^t}$, $b = 1 + r$

$1 + 0.023$

(a) $P(t) = 12000(1.023)^t$

(b) $P(3.5) = 12000(1.023)^{3.5}$

$= 12994$

(c) $\dfrac{20\cancel{000}}{12\cancel{000}} = \dfrac{\cancel{12000}(1.023)^t}{\cancel{12000}}$

$\dfrac{5}{3} = 1.023^t$

$\dfrac{\ln\left(\frac{5}{3}\right)}{\ln 1.023} = \dfrac{t \cdot \cancel{\ln 1.023}}{\cancel{\ln 1.023}}$

$t = \dfrac{\ln 5/3}{\ln 1.023} \approx 22.5$ years

$$0.8 = 0.985^t$$

$$\ln 0.8 = t \cdot \ln 0.985$$

$$t = \frac{\ln 0.8}{\ln 0.985} = 14.8 \text{ years}$$

(7) $P(t) = a \cdot b^t$ $\qquad$ $t = 0 \longleftrightarrow$ 5 yrs ago

$\underline{40000 = a} \cdot \cancel{b^0}$ $\qquad$ $(0, 40000)$

$$\frac{45500}{40000} = \frac{40000 \cdot b^5}{40000} \qquad (5, 45500)$$

$$\left(\frac{91}{80}\right)^{1/5} = \left(b^5\right)^{1/5}$$

$1.02610 = b$

(b)

$r = 0.02610$
$= 2.61\%$

$$\boxed{P(t) = 40000\,(\underbrace{1.02610}_{1+r})^t}$$

(c) $$\frac{50000}{40000} = \frac{\cancel{40000}\left(1.02610\right)^t}{\cancel{40000}}$$

$$\frac{5}{4} = 1.02610^t$$

$$\frac{\ln 5/4}{\ln 1.02610} = \frac{t \cdot \cancel{\ln 1.02610}}{\cancel{\ln 1.02610}}$$

$$t = \frac{\ln 5/4}{\ln 1.02610} \approx 8.66 \text{ years}$$

Half-Life Problem

Ex. Sodium-24 has a half-life of 14.96 hours. If we start with 10.0g, how long will it take to decay to 7.0g?

$$m(t) = \cancel{a}^{10} \cdot b^{t}$$

m = mass in g
t = time in hrs

plug in $\underset{(t,\ m)}{(14.96,\ 5)}$

$$5 = 10 \cdot b^{14.96}$$

$$\left(\tfrac{1}{2}\right)^{\frac{1}{14.96}} = \left(b^{14.96}\right)^{\frac{1}{14.96}}$$

$$b = \left(\frac{1}{2}\right)^{\frac{1}{14.96}} = 0.95472$$

[What is the rate of decay?

$$b = \underline{1 + r} = 0.95472$$

$$r = -0.04528$$

Mass decreases by 4.528% per hour.]

$$m(t) = 10(0.95472)^{t}$$

$$7 = 10(0.95472)^t$$

$$\frac{7}{10} = 0.95472^t$$

$$\ln 7/10 = t \cdot \ln 0.95472$$

$$t = \frac{\ln 7/10}{\ln 0.95472} = 7.70 \text{ hrs}$$

$\boxed{\#12}$ $m(t) = \frac{100}{\cancel{a}} \cdot b^t$ m in grams, t in days

plug in $\overset{t}{(10}, \overset{m}{89.2)}$

$$89.2 = 100\, b^{10}$$

$$(0.892)^{1/10} = (b^{10})^{1/10}$$

$$b = 0.892^{1/10} = 0.98864$$

$$\boxed{m(t) = 100(0.98864)^t}$$

$$\frac{50}{100} = \frac{\cancel{100}(0.98864)^t}{\cancel{100}}$$

$$\frac{1}{2} = 0.98864^t$$

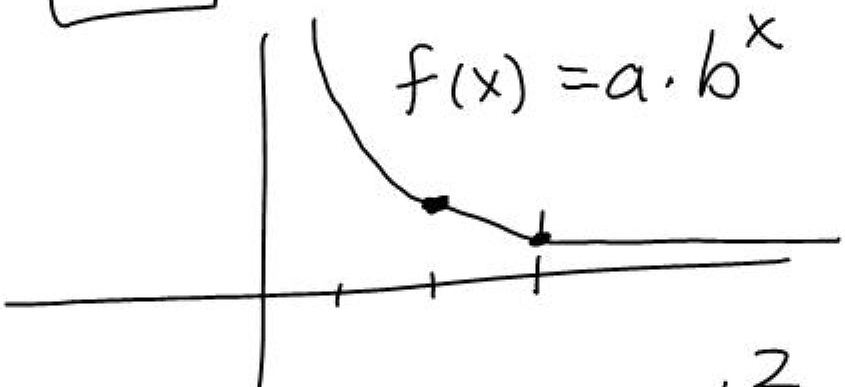
$$\ln \frac{1}{2} = t \cdot \ln 0.98864$$

$$t = \frac{\ln \frac{1}{2}}{\ln 0.98864} = 60.7 \text{ days}$$

$\boxed{\#1}$

(x, y)

$(2, \frac{4}{25})$ and $(3, \frac{4}{125})$



$$f(x) = a \cdot b^x$$

$$a = \frac{4}{25}\left(\frac{1}{5}\right)^{-2}$$

$$\boxed{a = 4}$$

solve by substitution

$$\frac{4}{25} = a \cdot b^2 \longrightarrow a = \frac{4}{25} b^{-2}$$

substitute

$$\frac{4}{125} = a \cdot b^3$$

$$\frac{4}{125} = \left(\frac{4}{25} b^{-2}\right) \cdot b^3 = \frac{4}{25} b^1$$

$$b = \frac{\cancel{25}}{\cancel{4}} \cdot \frac{\cancel{4}}{\cancel{125}_5} \longrightarrow \boxed{b = \frac{1}{5}}$$

$$\boxed{f(x) = 4\left(\frac{1}{5}\right)^x}$$

(c) $f(1) = \frac{4}{5}$

$$1 + r = \frac{1}{5}$$

$$r = -4/5 = -0.8$$

growth rate: -80%

HW #2, 8, 9, 13

#2

$(2,8) \rightarrow 8 = a \cdot b^2 \rightarrow a = 8b^{-2}$

$(5,3) \rightarrow 3 = a \cdot b^5$

$$3 = (8b^{-2})b^5$$

$$3 = 8b^3$$

$$\frac{3}{8} = b^3$$

$$\left(\frac{3}{8}\right)^{1/3} = b$$

$$\boxed{b = \frac{3^{1/3}}{2}}$$

$$a = 8b^{-2}$$

$$a = 8\left(\frac{3^{1/3}}{2}\right)^{-2}$$

$$a = 8 \cdot \left(\frac{2}{3^{1/3}}\right)^2$$

$$\boxed{a = \frac{32}{3^{2/3}}}$$

$$\boxed{f(x) = \frac{32}{3^{2/3}} \cdot \left(\frac{3^{1/3}}{2}\right)^x \\ f(x) \approx 15.3840\,(0.72112)^x}$$

(b) $1 + r = 0.72112$

$r = -0.27888$

$\boxed{\text{Growth rate} : -27.888\%}$

$\boxed{\#8}$ First, find an equation in the form

$m(t) = a \cdot b^t$, m in grams, t in days

$a = 100$

plug in $(330, 50)$ because 50 g remain after 1 half-life of 330 days

$50 = 100\,b^{330}$

$\frac{1}{2} = b^{330} \longrightarrow b = \left(\frac{1}{2}\right)^{\frac{1}{330}} = 0.99790$

$$\boxed{m(t) = 100(0.99790)^t}$$

$$1 + r = 0.99790$$

$$r = -0.0020982$$

$\boxed{\text{growth rate}: -0.20982\% \text{ per day}}$

(b) see above

(c) $m(365) = 100(0.99790)^{365}$

$= \underline{\underline{46.4\,g}}$

(d) $70 = 100(0.99790)^t$

$0.7 = 0.99790^t$

$\ln 0.7 = t \cdot \ln 0.99790$

$$\boxed{t = \frac{\ln 0.7}{\ln 0.99790} \approx 169.7 \text{ days}}$$

$\boxed{\#9}$ $W(t) = 4 \cdot b^t$

$2 = 4 \cdot b^{58}$ ← when $t = 58$, $\frac{1}{2}$ the initial amount remains

$1/2 = b^{58}$

$b = (1/2)^{1/58} = 0.98812$

$$\boxed{W(t) = 4(0.98812)^t}$$

$1 + r = 0.98812$

$$\boxed{r = -0.01188 = -1.188\% \text{ per min.}}$$

(b) see above

(c) $W(100) = 4(0.98812)^{100} = 1.21$ oz

(d) $3 = 4(0.98812)^t$

$3/4 = 0.98812^t$

$\ln 3/4 = t \cdot \ln 0.98812$

$t = \dfrac{\ln 3/4}{\ln 0.98812} \approx 24.1$ mins.

$\boxed{\#10}$ (a) $P(t) = 1800(1-0.15)^t$

$$P(t) = 1800(0.85)^t$$

(b) $P(5) = 1800(0.85)^5 = 799$ students

(c) $900 = 1800(0.85)^t$

$$1/2 = 0.85^t$$

$$\ln 1/2 = t \cdot \ln 0.85$$

$$t = \frac{\ln 1/2}{\ln 0.85} = 4.27 \text{ days}$$

$\boxed{\#13}$ (a) $m(t) = 1.00\, b^t$

$$0.9334 = b^{10}$$

$$b = (0.9334)^{\frac{1}{10}} = 0.99313$$

$$\boxed{m(t) = 0.99313^t}$$

(b) $0.5 = 0.99313^t$

$$\ln 0.5 = t \cdot \ln 0.99313$$

$$t = \frac{\ln 0.5}{\ln 0.99313}$$

$$t = 100.5 \text{ mins}$$

(c) Since it takes 100.5 mins for the quantity to decrease from 1.00 g to 0.500 g, the half-life is 100.5 mins