

#16. $x = -1$, $x = \pm 3i$

$x + 1 = 0$

$x^2 = (\cancel{3}i)^2$

$x^2 = -9$

$x^2 + 9 = 0$

$f(x) = (x + 1)(x^2 + 9)$

$= x^3 + 9x + x^2 + 9$

$= x^3 + x^2 + 9x + 9$

#18

$x = 3$, $x = 2 \pm 3i$

(choose one)

$x - 3 = 0$

$x = 2 - 3i$

$(x - 2)^2 = (-3i)^2$

FOIL $(x - 2)(x - 2) = (-1)^2 (3)^2 (i)^2$

$x^2 - 2x - 2x + 4 = (+1)(9)(-1)$

$x^2 - 4x + 4 = -9$

$x^2 - 4x + 13 = 0$

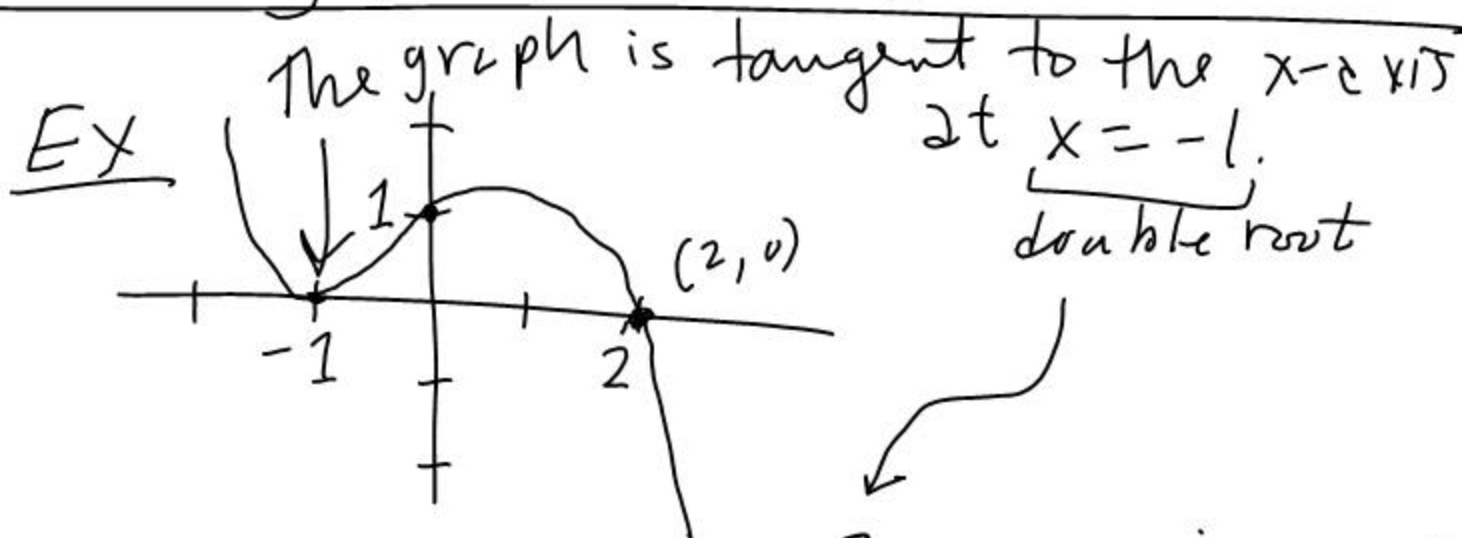
$$(\underline{x-3})(x^2 - 4x + 13)$$

$$x^3 - 4x^2 + 13x$$

$$-3x^2 + 12x - 39$$

$$f(x) = x^3 - 7x^2 + 25x - 39$$

Writing polynomials given a sketch



$$f(x) = a(x-2)(x+1)^2$$

← y-intercept

$$a(0-2)(0+1)^2 = 1$$

$$-2a = 1$$

$$a = -\frac{1}{2}$$

$$\begin{aligned}
 f(x) &= -\frac{1}{2}(x-2)(x+1)^2 && \underbrace{(x+1)(x+1)} \\
 &= -\frac{1}{2}(x-2)(x^2+2x+1) \\
 &= -\frac{1}{2}(\underbrace{x^3+2x^2+x}_{-2x^2-4x-2}) \\
 &= -\frac{1}{2}(x^3-3x-2) \\
 &= -\frac{1}{2}x^3 + \frac{3}{2}x + 1
 \end{aligned}$$

EX.  There is a flex point at $x=1$ (triple root)

$$\begin{aligned}
 f(x) &= a(x-1)^3(x+2) \\
 f(0) &= a(0-1)^3(0+2) = -3
 \end{aligned}$$

$$a(-1)(2) = -3$$

$$a = \frac{3}{2}$$

$$f(x) = \frac{3}{2}(x-1)^3(x+2)$$

$$(x-1)^3 = (x-1)(x-1)(x-1)$$

$$= (x-1)(x^2 - 2x + 1)$$

$$\begin{array}{r} x^3 - 2x^2 + x \leftarrow \text{distr } x \\ - x^2 + 2x - 1 \leftarrow \text{distr } -1 \end{array}$$

$$= x^3 - 3x^2 + 3x - 1$$

$$(x+2)(x^3 - 3x^2 + 3x - 1)$$

$$x^4 - 3x^3 + 3x^2 - x \leftarrow \text{distr. } x$$

$$2x^3 - 6x^2 + 6x - 2 \leftarrow \text{distr. } 2$$

$$x^4 - x^3 - 3x^2 + 5x - 2$$

$$f(x) = \frac{3}{2}x^4 - \frac{3}{2}x^3 - \frac{9}{2}x^2 + \frac{15}{2}x - 3$$

#25

$$y = a(x+1)^2(x-2)$$

$$a(0+1)^2(0-2) = -2 \quad \leftarrow y\text{-int}$$

$$a(1)(-2) = -2$$

$$a = 1$$

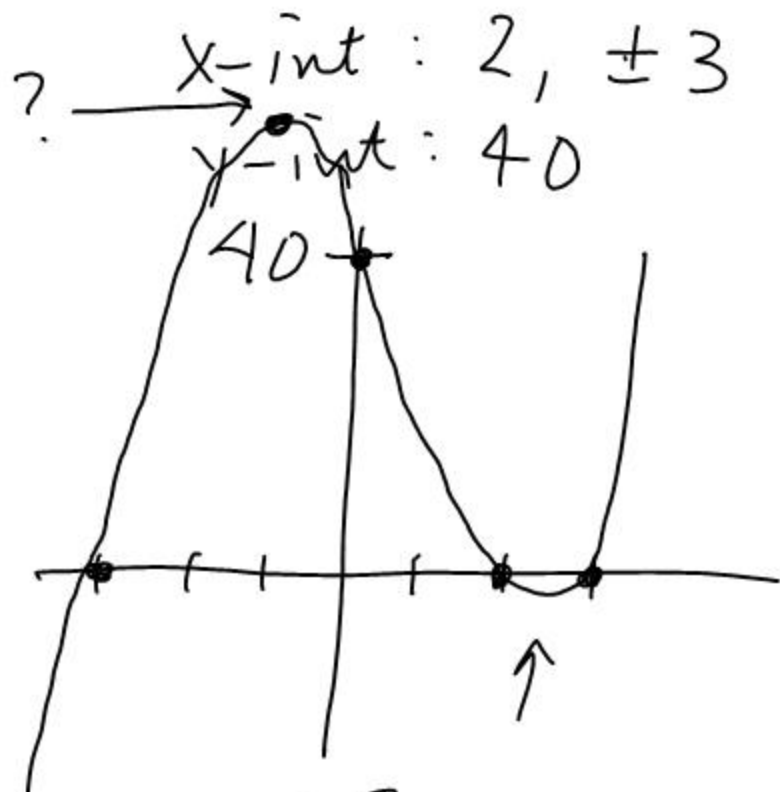
$$y = (x+1)^2(x-2)$$

$$= (x^2 + 2x + 1)(x-2)$$

$$= x^3 - 2x^2 + 2x^2 - 4x + x - 2$$

$$y = x^3 - 3x - 2$$

Sketching Polynomials



Ex
Sketch

$$y = x^4 - 5x^3 + 9x^2 - 7x + 2$$

$$\begin{array}{r|rrrrr}
 \downarrow & 1 & -5 & 9 & -7 & 2 \\
 & & 1 & -4 & 5 & -2 \\
 \hline
 & 1 & -4 & 5 & -2 & \boxed{0}
 \end{array}$$

$$(x-1)(x^3 - 4x^2 + 5x - 2)$$

$$\begin{array}{r|rrrr}
 \downarrow & 1 & -4 & 5 & -2 \\
 & & 1 & -3 & 2 \\
 \hline
 & 1 & -3 & 2 & \boxed{0}
 \end{array}$$

$$(x-1)^2 (x^2 - 3x + 2)$$

$$(x-1)^2 (x-1)(x-2)$$

$$y = (x-1)^3 (x-2)$$

HW 

28, 30 (finish previous problems
16-24 even)