Key

#37 $x^4 + 4x^3 + 6x^2 + 4x + 1$

Since all coefficients are positive, positive zeros aren't possible

-1	1	4	6	4	1
		-1	-3	-3	-1
	1	3	3	1	0

$(x+1)(x^3+3x^2+3x+1) = 0$

-1	1	3	3	1
		-1	-2	-1
	1	2	1	0

$(x+1)(x+1)(x^2+2x+1) = 0$

$(x+1)(x+1)(x+1)(x+1) = 0$

Zeros: $x = -1$ quadruple root

#38 $x^4+2x^3-13x^2-14x+24$

1 | 1 2 −13 −14 24
  | 1 3 −10 −24
  | 1 3 −10 −24 | 0

$(x-1)(x^3+3x^2-10x-24)=0$

X 1 | 1 3 −10 −24
  | 1 4 −6
  | 1 4 −6 | −30

X −1 | 1 3 −10 −24
  | −1 −2 12
  | 1 2 −12 | −12

X 2 | 1 3 −10 −24
  | 2 10 0
  | 1 5 0 | −24

−2 | 1 3 −10 −24
  | −2 −2 24
  | 1 1 −12 | 0

$(x-1)(x+2)(x^2+x-12)=0$ OVER

#38 continued

$(x-1)(x+2)(x-3)(x+4)=0$

zeros: 1, -2, 3, -4

#39 $x^5+4x^4-13x^3-52x^2+36x+144$

1	1	4	-13	-52	36	144
		1	5	-8	-60	-24
	1	5	-8	-60	-24	120

-1	1	4	-13	-52	36	144
		-1	-3	16	36	-72
	1	3	-16	-36	72	72

2	1	4	-13	-52	36	144
		2	12	-2	-108	-144
	1	6	-1	-54	-72	0

$(x-2)(x^4+6x^3-x^2-54x-72)=0$

2	1	6	-1	-54	-72
		2	16	30	-48
	1	8	15	-24	120

OVER

$$\begin{array}{r|cccc} -2 & 1 & 6 & -1 & -54 & -72 \\ & & -2 & -8 & 18 & 72 \\ \hline & 1 & 4 & -9 & -36 & 0 \end{array}$$

$$(x-2)(x+2)(x^3+4x^2-9x-36)=0$$

$$\begin{array}{r|cccc} -2 & 1 & 4 & -9 & -36 \\ & & -2 & -4 & -26 \\ \hline & 1 & 2 & 13 & -62 \end{array}$$

$$\begin{array}{r|cccc} 3 & 1 & 4 & -9 & -36 \\ & & 3 & 21 & 36 \\ \hline & 1 & 7 & 12 & 0 \end{array}$$

$$(x-2)(x+2)(x-3)(x^2+7x+12)=0$$

$$(x-2)(x+2)(x-3)(x+3)(x+4)=0$$

Zeros: $\pm 2, \pm 3, -4$

#40 $x^4 - 7x^3 + 13x^2 + 23x - 78$

$x = 1$

1	−7	13	23	−78
	1	−6	7	30
1	−6	7	30	−48

$x = -1$

1	−7	13	23	−78
	−1	8	−21	−2
1	−8	21	2	−80

$x = 2$

1	−7	13	23	−78
	2	−10	6	34
1	−5	3	17	−44

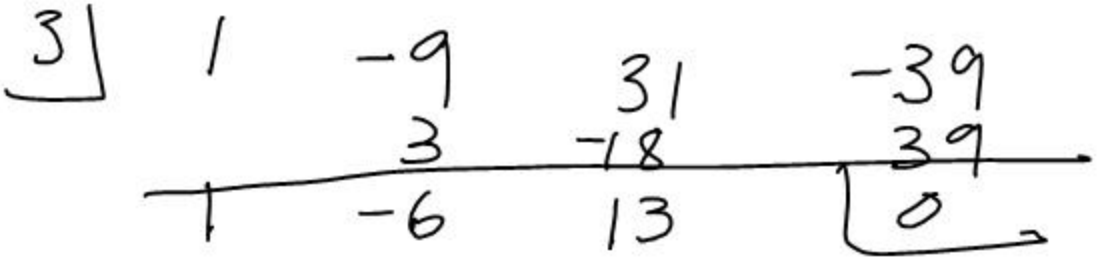
-2

1	−7	13	23	−78
	−2	18	−62	78
1	−9	31	−39	0

$(x+2)(x^3 - 9x^2 + 31x - 39) = 0$

$x = -2$

1	−9	31	−39
	−2	22	−106
1	−11	53	−145



$(x+2)(x-3)(x^2-6x+13)=0$

$x=-2$ $x=3$

Zeros

$$x=\frac{6\pm\sqrt{36-52}}{2}$$

$$x=\frac{6\pm\sqrt{-16}}{2}$$

$$x=\frac{6\pm 4i}{2}$$

$$x=3\pm 2i$$

#41

$x^4+10x^3+22x^2-40x-104$

$x=1$

	1	10	22	−40	−104
		1	11	33	−7
	1	11	33	−7	−111

$x=-1$

	1	10	22	−40	−104
		−1	−9	−13	53
	1	9	13	−53	−51

2

	1	10	22	−40	−104
		2	24	92	104
	1	12	46	52	0

$(x-2)(x^3+12x^2+46x+52)=0$

$x=2$

	1	12	46	52
		2	28	148
	1	14	74	200

-2

	1	12	46	52
		−2	−20	−52
	1	10	26	0

Ans

#41 continued

$(x-2)(x+2)(x^2+10x+26)=0$

$x=2$ | $x=-2$

$x=\dfrac{-10\pm\sqrt{100-104}}{2}$

$x=\dfrac{-10\pm 2i}{2}$

zeros

$x=-5\pm i$

#42 x^6+7x^3-8

$(x^3+8)(x^3-1)$

Look! A sum of cubes and a difference of cubes!

$(x+2)(x^2-2x+4)(x-1)(x^2+x+1)=0$

$x=-2$

$x=\dfrac{2\pm\sqrt{4-16}}{2}$

$x=\dfrac{2\pm 2i\sqrt{3}}{2}$

$x=1\pm i\sqrt{3}$

$x=1$

$x=\dfrac{-1\pm\sqrt{1-4}}{2}$

$x=\dfrac{-1\pm i\sqrt{3}}{2}$

zeros