

Factor
Ex. $27 - x^3$

$$3^3 - x^3$$

$$(3 - x)(9 + 3x - x^2)$$

$$27 - x^3 = 0$$

$$(3 - x)(9 + 3x - x^2) = 0$$

$$3 - x = 0 \quad \text{or} \quad 9 + 3x - x^2 = 0$$

$$\boxed{x = 3}$$

$$9 - 4(-9)$$

$$9 - (-36)$$

$$45$$

$$9 \cdot 5$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(-1)(9)}}{2(-1)}$$

$$x = \frac{-3 \pm 3\sqrt{5}}{-2}$$

Ex Factor: $y^3 + 125$ $y^3 + 5^3$

$$(y + 5)(y^2 - 5y + 25)$$

#6 $f(x) = x^4 - x = 0$

$$x(x^3 - 1) = 0$$

$$x(x - 1)(x^2 + x + 1) = 0$$

$x = 0$ or $x - 1 = 0$ or $x^2 + x + 1 = 0$

$x = 1$ $x = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)}$

$$\sqrt{-3} = \sqrt{(-1)(3)}$$

↓ ↓
i $\sqrt{3}$

$$x = \frac{-1 \pm \sqrt{3}i}{2}$$

#12

$$f(x) = \underbrace{x^3 + 2x^2}_{x^2(x+2)} - \underbrace{4x - 8}_{4(x-2)} = 0$$

$$x^2(\cancel{x+2}) - 4(\cancel{x-2})$$

$$(x+2)(x^2-4) \leftarrow x^2-2^2$$

$$(x+2)(x-2)(x+2) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ \boxed{x = -2} & \boxed{x = 2} \end{array}$$

double
root

Guess + check to find zeros

$$f(x) = x^3 + x^2 - 21x - 45 \quad \pm 1$$

$$\text{guess 1: } 1^3 + 1^2 - 21 - 45 \neq 0 \quad \neq 3$$

$$\text{guess -1: } (-1)^3 + (-1)^2 - 21(-1) - 45 \quad \neq 5$$

$$\cancel{x} + \cancel{x} + 21 - 45 \neq 0 \quad \neq 9$$

$$\neq 15$$

$$\neq 45$$

guess 3 $3^3 + 3^2 - 21(3) - 45$

$27 + 9 - 63 - 45 \neq 0$

guess -3 $(-3)^3 + (-3)^2 - 21(-3) - 45 = 0$
 $-27 + 9 + 63 - 45 = 0$
 $\frac{9}{72} \quad \frac{27}{-72}$

synthetic substitution

(a way to plug numbers into a polynomial)

guess \downarrow
3 | 1 1 -21 -45
 | 3 12 -27

 | 1 4 -9 -72 = f(3)

\downarrow
-3 | 1 1 -21 -45
 | -3 6 45

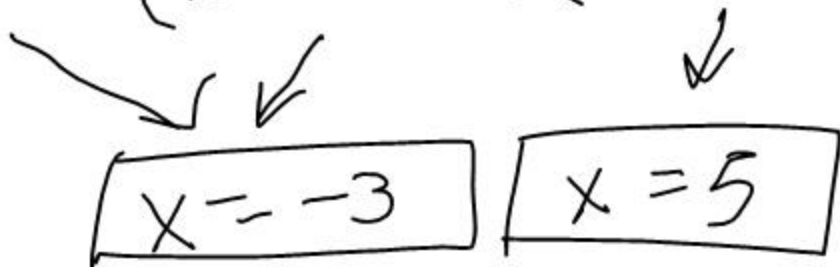
 | 1 -2 -15 0 = f(-3)

$(x+3) (x^2 - 2x - 15)$

$$x^3 + x^2 - 21x - 45 = 0$$

$$(x+3)(x^2 - 2x - 15) = 0$$

$$(x+3)(x+3)(x-5) = 0$$


$$\boxed{x = -3} \quad \boxed{x = 5}$$

$$\begin{array}{r} \underline{\underline{1}} \quad 1 \quad 4 \quad -60 \quad -256 \quad -256 \\ \phantom{\underline{\underline{1}}} \quad \quad 1 \quad 5 \quad 55 \quad -201 \\ \hline 1 \quad 5 \quad -55 \quad -201 \end{array}$$

$$\begin{array}{r} \underline{\underline{-1}} \quad 1 \quad 4 \quad -60 \quad -256 \quad -256 \\ \phantom{\underline{\underline{-1}}} \quad \quad -1 \quad -3 \quad 63 \quad 193 \\ \hline 1 \quad 3 \quad -63 \quad -193 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & 4 & -60 & -256 & -256 \\ & & 2 & 12 & -96 & \\ \hline & 1 & 6 & -48 & -352 & \end{array}$$

$$\begin{array}{r|rrrrr} -2 & 1 & 4 & -60 & -256 & -256 \\ & & -2 & -4 & 128 & 256 \\ \hline & 1 & 2 & -64 & -128 & 0 \end{array}$$

cubic \rightarrow

$$(x+2) (x^3 + 2x^2 - 64x - 128)$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -64 & -128 \\ & & -2 & 0 & 128 \\ \hline & 1 & 0 & -64 & 0 \end{array}$$

$$(x+2)(x+2)(x^2-64) = 0$$

$$(x+2)^2(x-8)(x+8) = 0$$

$$\boxed{x = -2} \quad \boxed{x = \pm 8}$$

double
root

HW

#1-19 odd

#34, 37