Review : Integral Calculus

① $\int (3x-4)\,dx = \frac{3}{2}x^2 - 4x + C$

② $\int \left(3x^4 - \frac{1}{x}\right)dx = \frac{3}{5}x^5 - \ln|x| + C$

③ $\int \frac{dx}{x^2} = \int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = -x^{-1} + C$

or $-\frac{1}{x} + C$

Reminder: Don't use a log for the antiderivative for every problem with x in the denominator!

④ $\int \cos x\,dx = \sin x + C$

⑤ $\frac{1}{2}\int 2e^{2x-3}\,dx = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C$

$= \frac{1}{2}e^{2x-3} + C$

$u = 2x-3$

$du = 2\,dx$

(6) $\frac{1}{4}\int 4x^3 \cdot (x^4-1)^5\,dx = \frac{1}{4}\int u^5\,du$

$u = x^4 - 1$

$du = 4x^3\,dx$

$= \frac{1}{24}u^6 + C = \frac{1}{24}(x^4-1)^6 + C$

(7) $\frac{1}{4}\int \frac{4}{4x+1}\,dx = \frac{1}{4}\int \frac{du}{u} = \frac{1}{4}\ln|u| + C$

$u = 4x+1$

$du = 4\,dx$

$= \frac{1}{4}\ln|4x+1| + C$

(8) $\frac{1}{4}\int \frac{4}{(4x+1)^2}\,dx = \frac{1}{4}\int \frac{du}{u^2} = \frac{1}{4}\int u^{-2}\,du$

$u = 4x+1$

$du = 4\,dx$

$= -\frac{1}{4}u^{-1} + C$

$= \frac{-1}{4(4x+1)} + C$

Note Make sure to compare #7 and #8 and understand why one ends up with a log and the other doesn't.

(9) $\frac{1}{2}\int 2x \cdot \sin(x^2)\,dx = \frac{1}{2}\int \sin u\,du$

$u = x^2$

$du = 2x\,dx$

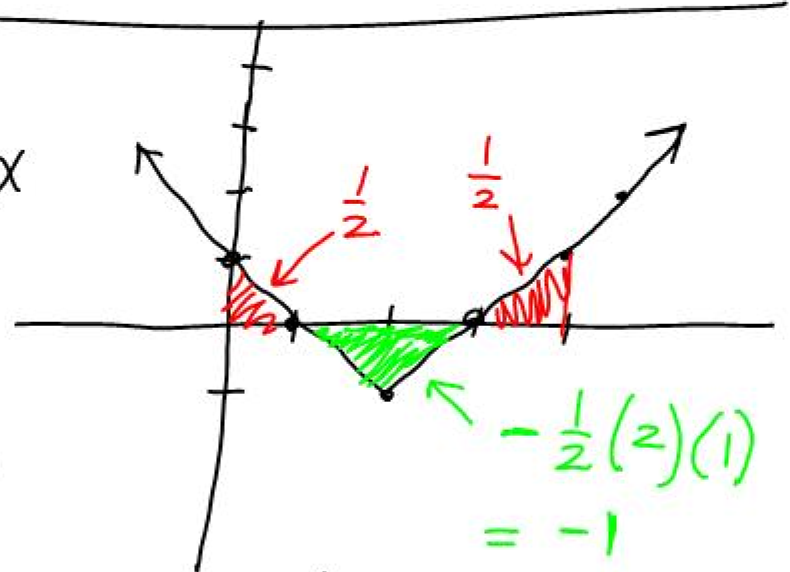
$= \frac{-1}{2}\cos u + C = -\frac{1}{2}\cos(x^2) + C$

(10) omit

(11) $\int_0^4 \left(|x-2| - 1\right) dx$

right 2

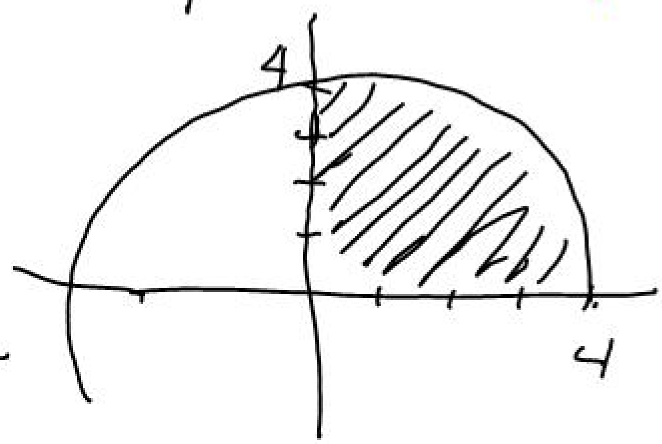
down 1



$= \frac{1}{2} - 1 + \frac{1}{2} = 0$

(12) $\int_0^4 \sqrt{16 - x^2}\,dx$

$= \frac{1}{4} \cdot \pi(4)^2 = 4\pi$



(13) $\int_0^{\pi/2} \cos x \, dx = \left[\sin x\right]_0^{\pi/2}$

$= \sin\frac{\pi}{2} - \sin 0 = 1 - 0 = 1$

(14) $\int_{\ln 2}^{\ln 3} e^x \, dx = \left[e^x\right]_{\ln 2}^{\ln 3} = e^{\ln 3} - e^{\ln 2}$

$= 3 - 2$

$= 1$

(15) $\int_1^{e^3} \frac{1}{x} \, dx = \left[\ln x\right]_1^{e^3}$

$= \ln e^3 - \ln 1$

$= 3 - 0 = 3$

(16) $\frac{-1}{2}\int_0^4 -2x \cdot \sqrt{16-x^2} \, dx = -\frac{1}{2}\int_{16}^{0} u^{1/2} \, du$

$u = 16 - x^2$

$u = 16 - 4^2 = 0$

$u = 16 - 0^2 = 16$

$du = -2x\,dx$

$= \left[-\frac{1}{3}u^{3/2}\right]_{16}^{0}$

$= -\frac{1}{3}(0)^{3/2} - \frac{-1}{3}(16)^{3/2} = \frac{1}{3}(4)^3 = \frac{64}{3}$

(17) $\frac{1}{2}\int_0^1 \frac{2x}{x^2+4}\,dx = \frac{1}{2}\int_4^5 \frac{du}{u} = \left[\frac{1}{2}\ln u\right]_4^5$

$u = 1^2+4 = 5$ $\quad u = x^2+4$ $\quad = \frac{1}{2}\ln 5 - \frac{1}{2}\ln 4$

$u = 0^2+4 = 4$ $\quad du = 2x\,dx$

<u>or</u> $\frac{1}{2}\ln\frac{5}{4}$

(18) $\frac{1}{2}\int_0^{\pi/6} 2\cos(2x)\,dx = \frac{1}{2}\int_0^{\pi/3}\cos u\,du$

$u = 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$ $\quad u = 2x$ $\quad = \left[\frac{1}{2}\sin u\right]_0^{\pi/3}$

$du = 2\,dx$

$u = 2(0) = 0$

$= \frac{1}{2}\sin\frac{\pi}{3} - \frac{1}{2}\sin 0 = \frac{\sqrt{3}}{4}$

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(19) $a(t) = 2t-7$, $v(0) = 10$, $s(0) = -3$

(a) $v(t) = \int (2t-7)\,dt = t^2 - 7t + C$

$v(0) = 0^2 - 7(0) + C = 10 \rightarrow C = 10$

$\boxed{v(t) = t^2 - 7t + 10}$

$v(3) = 3^2 - 7(3) + 10 = -2$ m/s

The velocity is negative, so the particle is moving to the <u>left</u>.

(b) $s(t) = \int (t^2 - 7t + 10)\,dt$

$= \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + C$

$s(0) = \frac{1}{3}(0)^3 - \frac{7}{2}(0)^2 + 10(0) + C = -3$

$C = -3$

$$\boxed{s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t - 3}$$

$s(3) = \frac{1}{3}(3)^3 - \frac{7}{2}(3)^2 + 10(3) - 3 = \frac{9}{2}$ m

(c) $a(3) = 2(3) - 7 = -1$ (force pulling to the left)

$v(3) = -2$ (moving to the Left)

The particle's speed is increasing

(d) The particle is at rest when its velocity is zero. $v(t) = t^2 - 7t + 10 = 0$

$(t-5)(t-2) = 0$

$$\boxed{t = 2 \text{ secs or } 5 \text{ secs}}$$

(e) $\int_0^{10} |t^2 - 7t + 10|\, dt = 92.3\,m$

(f) $\int_0^{10} (t^2 - 7t + 10)\, dt = 83.3\,m$

(20) $v(t) = \cos\frac{t}{2}$, $0 \le t \le 2\pi$, $s(0) = 4$

(a) $s(t) = 2\int \frac{1}{2}\cos\frac{t}{2}\, dt = 2\int \cos u\, du$

$u = \frac{t}{2}$ $\quad = 2\sin u + C$

$du = \frac{1}{2}dt$ $\quad = 2\sin\frac{t}{2} + C$

$s(0) = 2\sin\frac{0}{2} + C = 4 \rightarrow C = 4$

$$\boxed{s(t) = 2\sin\left(\frac{t}{2}\right) + 4}$$

$s\left(\frac{2\pi}{3}\right) = 2\sin\left(\frac{2\pi/3}{2}\right) + 4 = 2\sin\left(\frac{\pi}{3}\right) + 4$

$= 2\left(\frac{\sqrt{3}}{2}\right) + 4$

$= \underline{(\sqrt{3} + 4)\,m}$

(b) $v(t) = \cos\frac{t}{2} = 0$

$$\frac{t}{2} = \frac{\pi}{2} \rightsquigarrow t = \underline{\underline{\pi \text{ secs}}}$$

(c) $a(t) = \frac{d}{dt}\left[\cos\frac{t}{2}\right] = -\frac{1}{2}\sin\frac{t}{2}$

$$a\left(\frac{2\pi}{3}\right) = -\frac{1}{2}\sin\left(\frac{2\pi/3}{2}\right) = -\frac{1}{2}\sin\left(\frac{\pi}{3}\right)$$

$$= -\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) \ \underline{\underline{\text{neg}}}$$

$$v\left(\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi/3}{2}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \ \underline{\underline{\text{pos}}}$$

$\boxed{\text{The particle's speed is decreasing.}}$