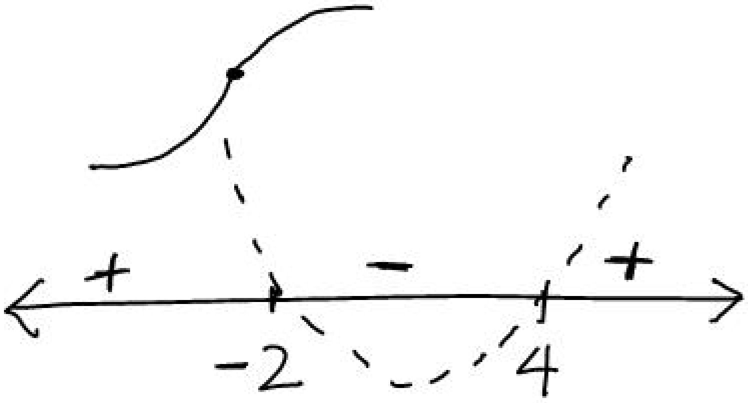
$\boxed{7.5}$ #8

$f''(x)$:

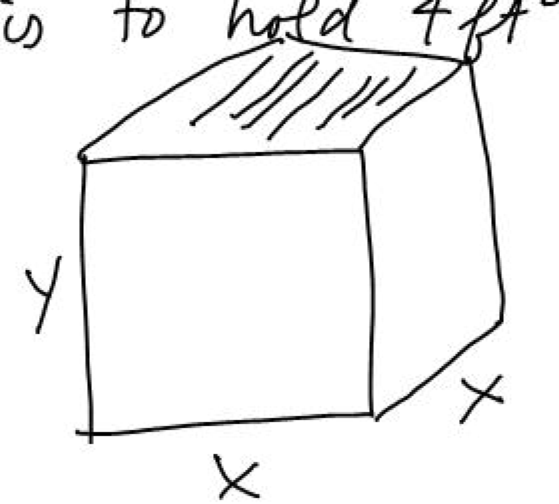


f is concave up on $(-\infty, -2) \cup (4, \infty)$

points of inflexion at $x = -2,\ 4$

Optimization

Ex. An open-top rectangular box with a square bottom is to hold $4\ ft^3$ of volume. Find the dimensions that have the least surface area.



Surface area: $A = x^2 + 4yx$

Find the minimum

volume: $x^2 y = 4$

$y = \frac{4}{x^2}$

$A = x^2 + 4\left(\frac{4}{x^2}\right) \cdot x$

$A = x^2 + \frac{16}{x}$ find the minimum

$A = x^2 + 16x^{-1}$

$A' = 2x - 16x^{-2}$

$A' = 2x - \frac{16}{x^2} = 0$

$\frac{0 - 16}{x^2}$

mult by x^2 on both sides

$2x^3 - 16 = 0$

$2x^3 = 16$

$x^3 = 8$

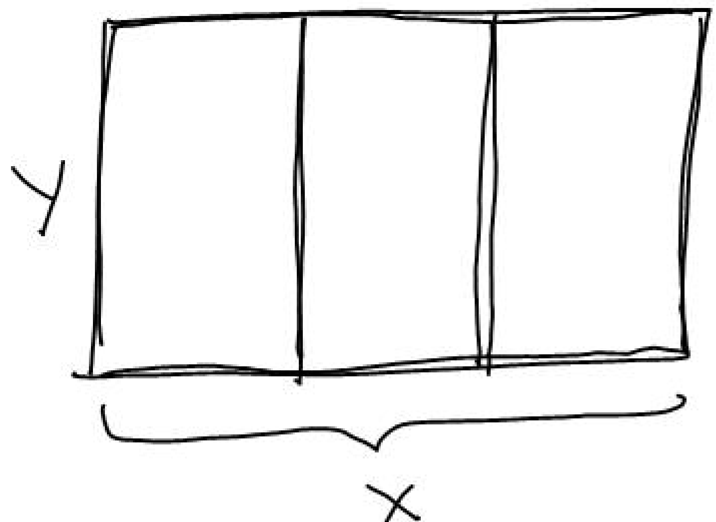
$\boxed{x = 2 \text{ ft}}$

$y = \frac{4}{2^2}$

$\boxed{y = 1 \text{ ft}}$

Ex. A rancher has 1000 ft of fence material to build a corral as shown:

Find the dimensions of the corral with the largest area.



$$A = xy \leftarrow \text{maximize}$$

$$2x + 4y = 1000$$

$$4y = 1000 - 2x$$

$$y = 250 - \tfrac{1}{2}x$$

$$A = x\left(250 - \tfrac{1}{2}x\right)$$

$$A = 250x - \tfrac{1}{2}x^2 \leftarrow \text{maximize}$$

$\leftarrow$ parabola that opens down

$$A' = 250 - x = 0$$

$$\boxed{x = 250 \text{ ft}}$$

$$\boxed{y = 125 \text{ ft}}$$

Ex. Find the x-value of the vertex of $f(x) = ax^2 + bx + c$

$$f'(x) = 2ax + b = 0$$

$$2ax = -b$$

$$\frac{-b \cancel{\pm\sqrt{b^2-4ac}}}{2a} \qquad \boxed{x = \frac{-b}{2a}}$$

$\boxed{TX}$ #1 $\quad x = 1^{st}$ number

$y = 2^{nd}$ number

$$z = x + \sqrt{y} \qquad x + y = 20$$

$$y = 20 - x$$

$$z = x + \sqrt{20 - x}$$

$$z' = 1 + \frac{1}{2}(20-x)^{-\frac{1}{2}}(-1) = 1 - \frac{1}{2\sqrt{20-x}}$$

$$z' = 1 - \frac{1}{2\sqrt{20-x}} = 0$$

$$z' = 1 - \frac{1}{2\sqrt{20-x}} = 0$$

$$\frac{1}{2\sqrt{20-x}} = 1$$

$$1 = 2\sqrt{20-x}$$

$$\frac{1}{2} = \sqrt{20-x}$$

$$\frac{1}{4} = 20 - x$$

$$x = 19.75$$