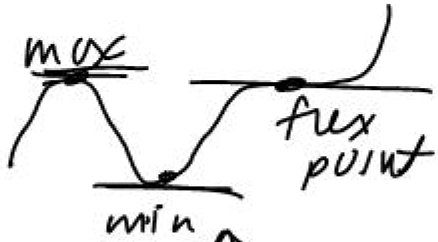
$\boxed{\text{TR}}$ **#3** $f(x) = x^{5/3}$



$$f'(x) = \frac{5}{3}x^{2/3} = 0$$

critical value → $x = 0$

$f'(x)$: + | +

f is incr. 0 f is incr.

↑

flex point

(∴ no extrema)

$$f''(x) = \frac{10}{9}x^{-1/3} = \frac{10}{9x^{1/3}} = 0$$

hypercritical value: $f''(0)$ dne

$f''(x)$: − | +

f is concave down 0 f is concave up

↑

point of inflection

(flex pt.)

change in concavity →

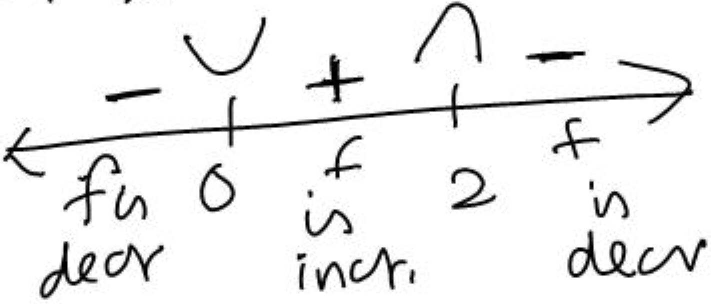
$\boxed{7R}$ #6. $f(x) = x^2 e^{-x}$

$$f'(x) = \underbrace{\overset{\downarrow}{x^2} \cdot \overset{\downarrow}{e^{-x}} \cdot (-1)} + \underbrace{\overset{\downarrow}{e^{-x}} \cdot \overset{\downarrow}{2x}}$$

$$= x\underbrace{e^{-x}}_{\text{pos.}}\left(\underline{2 - x}\right) = 0$$

$x = 0$ or $x = 2$ ← critical values

$f'(x)$:



min at $x = 0$; max at $x = 2$

$(0,0)$ $\left(2, \frac{4}{e^2}\right)$

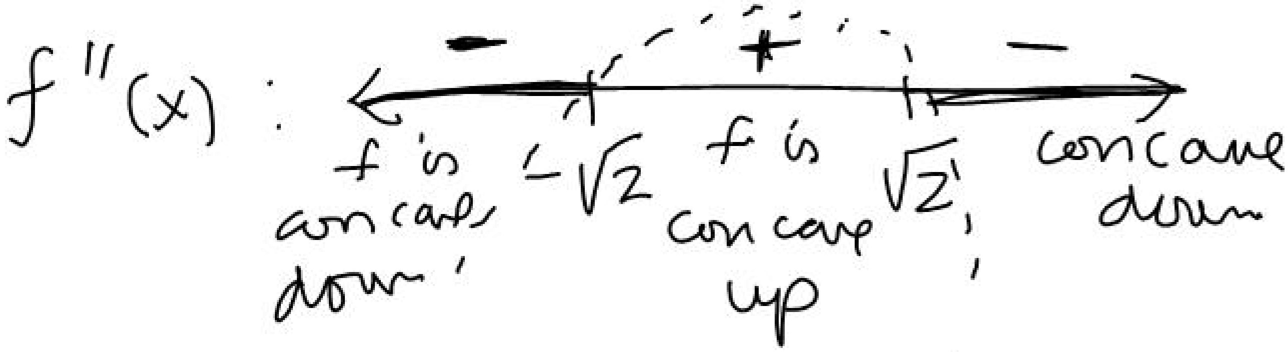
$$f''(x) = \underline{x^2 \cdot e^{-x}(-1)} + \underline{e^{-x} \cdot 2x} + \underline{e^{-x}(2)} + \underline{2x\, e^{-x}(-1)}$$

$$= e^{-x}\left(-x^2 + \cancel{2x} + 2 - \cancel{2x}\right)$$

$$= \underbrace{e^{-x}}_{\text{pos}}\left(2 - x^2\right) = 0 \Rightarrow x = \pm\sqrt{2}$$

$x^2 = 2$

hypercritical values



points of inflexion at $\left(-\sqrt{2}, 2e^{\sqrt{2}}\right)$

$\left(\sqrt{2}, \frac{2}{e^{\sqrt{2}}}\right)$