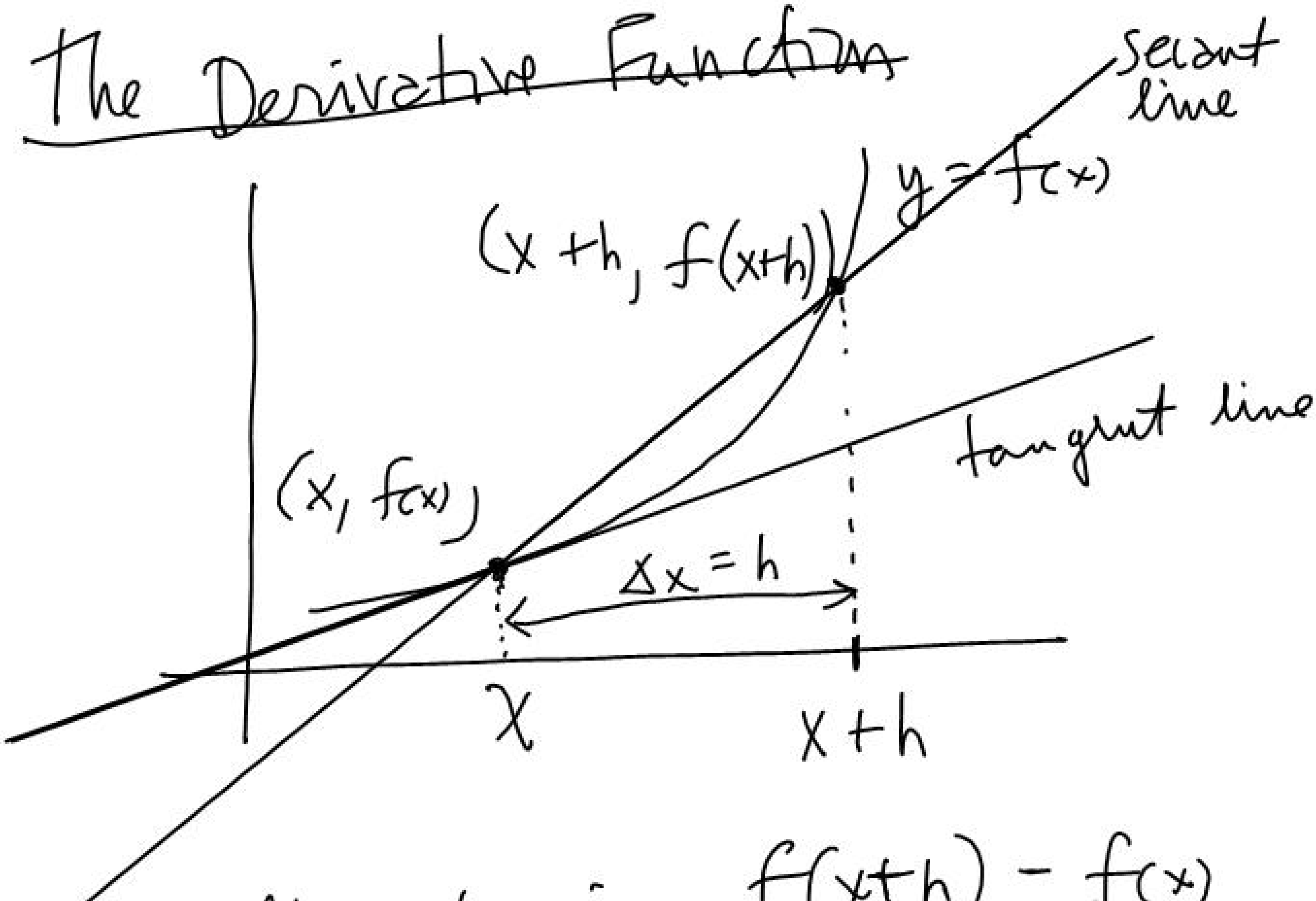
The Derivative Function



Secant line slope is $\dfrac{f(x+h) - f(x)}{h}$

tangent line slope (the derivative function)

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

The derivative at $x = a$.

Ex. Find the derivative function for $f(x) = 4 - x^2$. $f(x+h) = 4 - (x+h)^2$

$$f'(x) = \lim_{h \to 0} \frac{[4 - (x+h)^2] - [4 - x^2]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4} - \cancel{x^2} - 2xh - h^2 - \cancel{4} + \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-2x - h)}{\cancel{h}} = -2x$$

$$\boxed{f(x) = \underline{4 - x^2} \Rightarrow f'(x) = -2x}$$

Evaluate $f'(2)$

$$f'(2) = -2(2) = -4$$

Ex. Where is the vertex of

$$f(x) = 4 - 3x - 5x^2 \ ?$$

$$f(x) = 4 - 3(x+h) - 5(x+h)^2$$

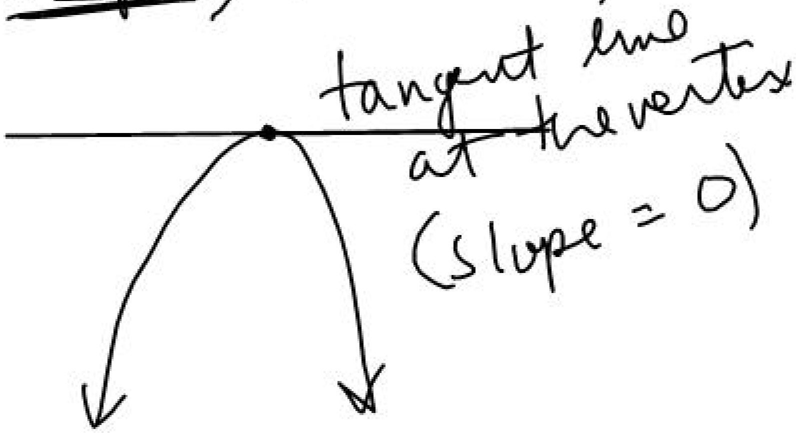
step 1) find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{\left[4 - 3(x+h) - 5\underbrace{(x+h)^2}_{(x^2+2xh+h^2)}\right] - \left[4 - 3x - 5x^2\right]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4} - \cancel{3x} - 3h - \cancel{5x^2} - 10xh - 5h^2 - \cancel{4} + \cancel{3x} + \cancel{5x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-3 - 10x - 5h)}{\cancel{h}} = -3 - 10x$$

step 2) Find where the vertex occurs



$$-3 - 10x = 0$$

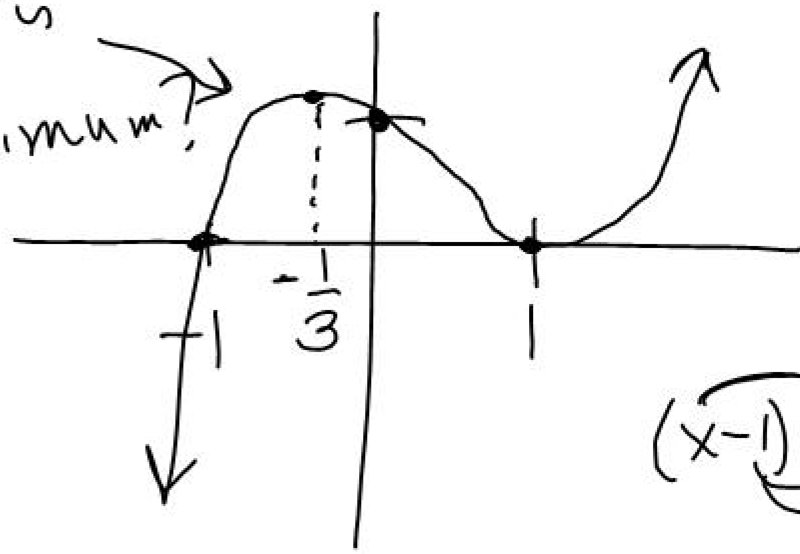
$$-10x = 3$$

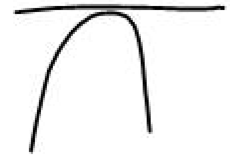
$$x = -\frac{3}{10}$$

$(0-1)^2(0+1)=1$

Ex. Sketch $f(x)=(x-1)^2(x+1)$

Where's this maximum?





$(x-1)(x-1)$

Step 1) Find $f'(x)$.

$$f(x)=\underline{(x-1)^2}(x+1) = (x+1)(x^2-2x+1)$$

$$= x^3-2x^2+x+x^2-2x+1$$

$$f(x)=x^3-x^2-x+1$$

$$f'(x)=\lim_{h\to 0}\frac{\left[(x+h)^3-(x+h)^2-(x+h)+1\right]-\left[x^3-x^2-x+1\right]}{h}$$

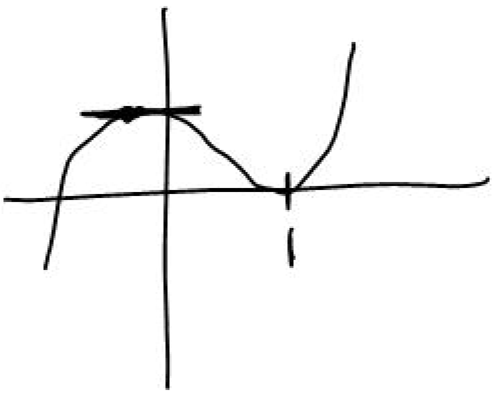
$$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h^1+3x^1h^2+h^3-\cancel{x^2}-2xh-h^2-\cancel{x}-h}{h}$$

$$\ldots\ \frac{+\cancel{1}-\cancel{x^3}+\cancel{x^2}+\cancel{x}-\cancel{1}}{}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(3x^2 + 3xh + h^2 - 2x - h - 1\right)}{\cancel{h}}$$

$$f'(x) = 3x^2 - 2x - 1$$

<u>step 2)</u> Find the x-coordinate of the max



$$3x^2 - 2x - 1 = 0$$

$$(3x+1)(x-1) = 0$$

$$\boxed{\underbrace{x = -\tfrac{1}{3}}_{\text{max here}} \text{ or } \underbrace{x = 1}_{\text{min here}}}$$

Ex. Find $f'(x)$ for $f(x) = \sqrt{2x+1}$

$f'(x) =$

$$\lim_{h \to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}$$

$$\cancel{2x} + 2\cancel{h} + \cancel{1} - \cancel{2x} - \cancel{1}$$

$$= \lim_{h \to 0} \frac{2(x+h) + 1 - (2x+1)}{\cancel{h}\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)}$$

$$\sqrt{2(x+0)+1} + \sqrt{2x+1}$$

$$f'(x) = \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$