

Calculus: Limits

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-6x+8} = \lim_{x \rightarrow 2} \frac{\cancel{x-2}}{(\cancel{x-2})(x-4)} = \frac{1}{-2}$$

$$2^2 - 6(2) + 8 = 0$$

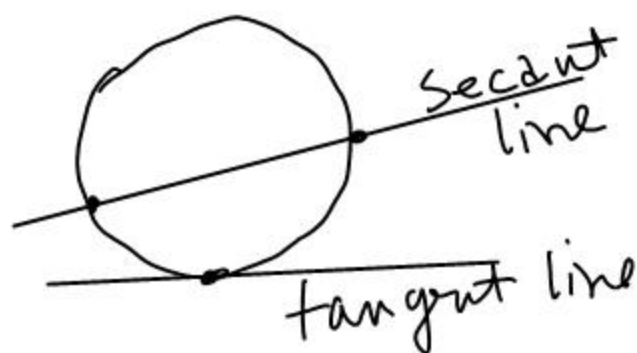
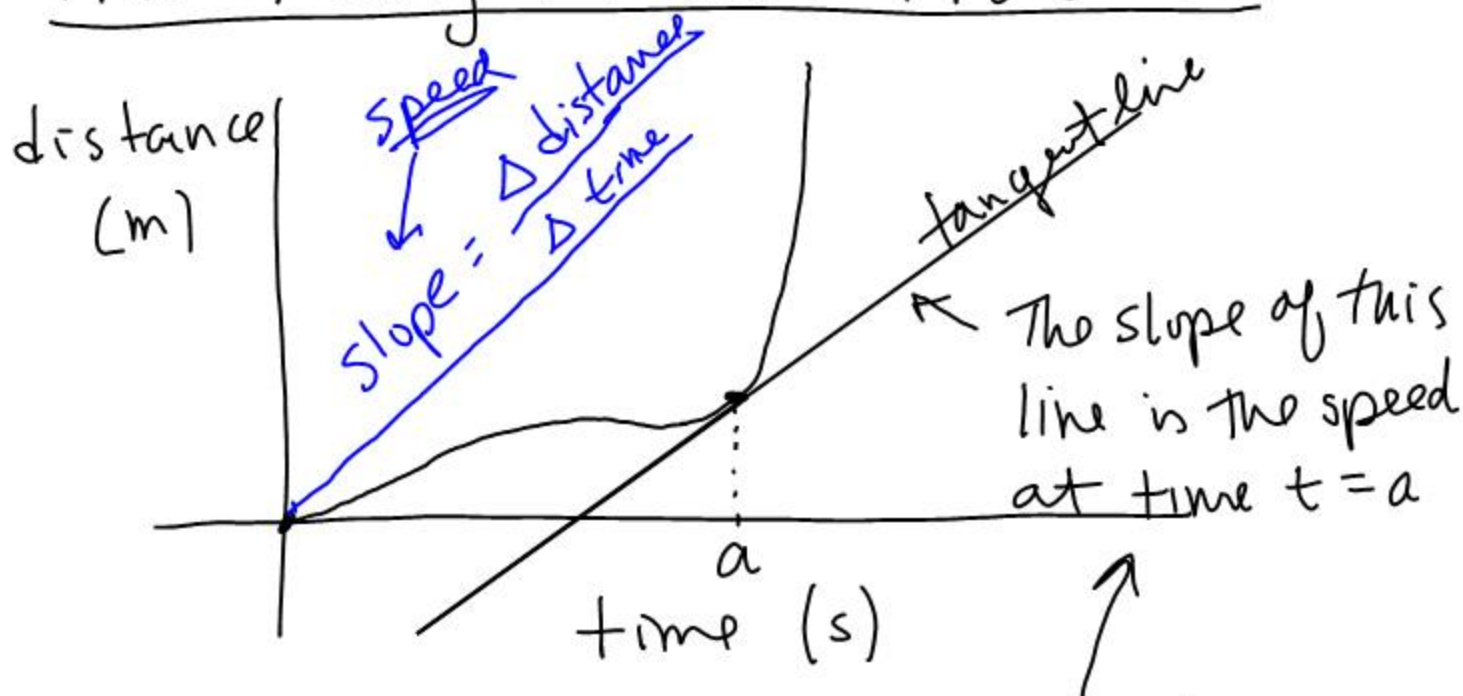
$$= -\frac{1}{2}$$

$$\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x-4} \cdot \frac{\sqrt{x+5} + 3}{\sqrt{x+5} + 3}$$

$$= \lim_{x \rightarrow 4} \frac{\cancel{x-4} \cdot 9}{(\cancel{x-4})(\sqrt{x+5} + 3)} = \frac{1}{6}$$

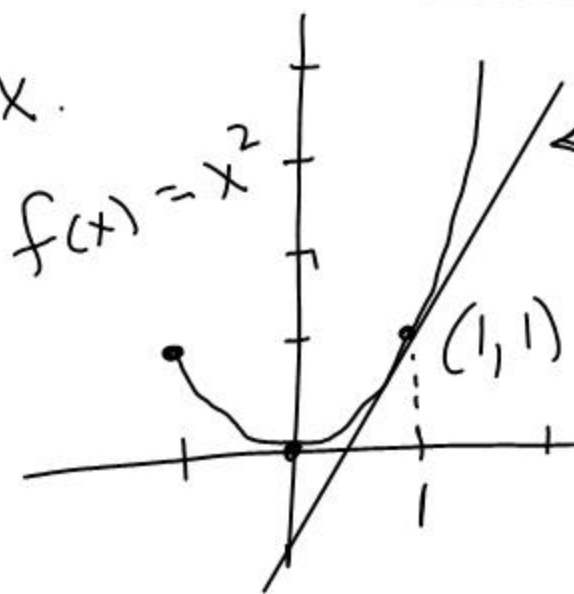
$\sqrt{4+5} + 3$

The Tangent Line Problem



the instantaneous speed

Ex.



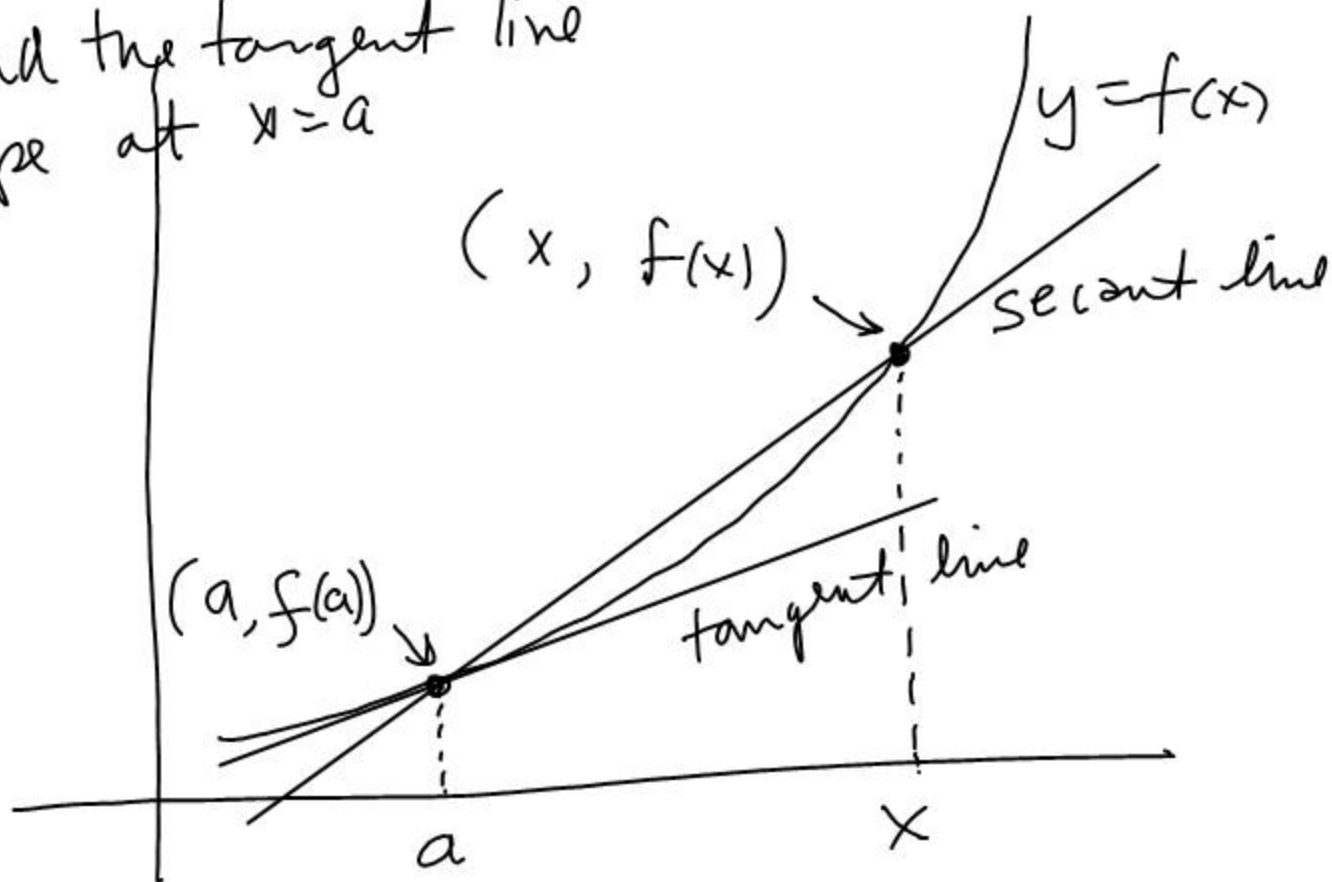
Find the slope of this tangent line

$$f'(1) = \lim_{x \rightarrow 1} \frac{x^2 - 1^2}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$

$$= 2$$

Find the tangent line
slope at $x=a$



secant line slope : $\frac{f(x) - f(a)}{x - a}$

tangent line slope :

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

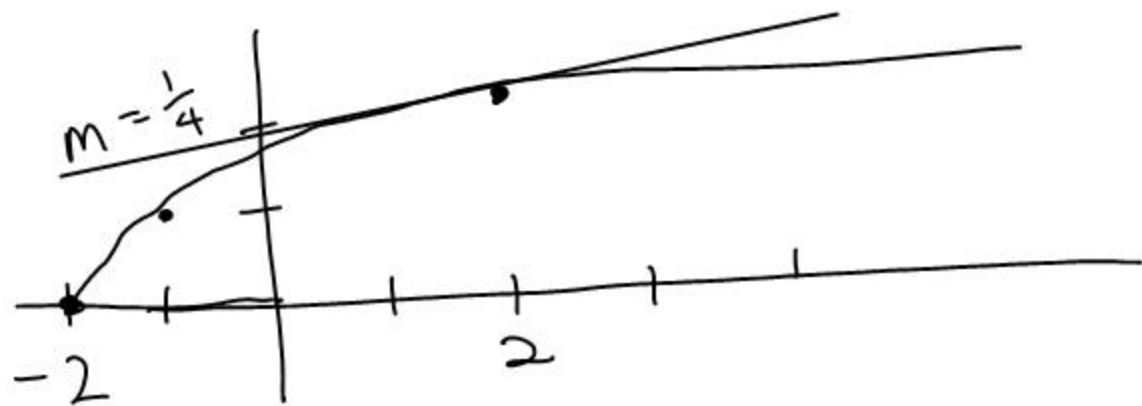
↑
The derivative of f at $x=a$

Ex. Find $f'(2)$ for $f(x) = \sqrt{x+2}$

$$f'(2) = \lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{x-2} \cdot \frac{\sqrt{x+2} + 2}{\sqrt{x+2} + 2}$$

$f(2) = \sqrt{2+2} = 2$

$$= \lim_{x \rightarrow 2} \frac{\cancel{x+2} - 4}{(x-2)(\sqrt{x+2} + 2)} = \frac{1}{4}$$



$$\left. \frac{d}{d \boxed{x}} (\sqrt{\boxed{x+2}}) \right|_{\boxed{x} = \boxed{2}}$$