$\boxed{15J}$ #5

X = mass of powder

$X \sim N(500, 20^2)$

a) $P(X<475) = P(Z<-1.25) =$ normalcdf $(-9,-1.25)$

$Z = \frac{475-500}{20} = -1.25$

$= 0.106$

b) $(0.106)(0.106)(0.106) = 0.00119$

$\boxed{15M}$ #5 $N(136, \sigma^2)$

$P(X>145) = 0.12 \leftarrow 88^{th}$ percentile

$Z =$ invnorm $(0.88) = 1.175 = \frac{145-136}{\sigma}$

$\boxed{\sigma = 7.66 \text{ cm}}$

$\boxed{15M \ \#10}$

$P(X < 108) = 0.3 \leftarrow$ 30th percentile

$P(X > 154) = 0.2 \leftarrow$ 80th percentile

$$Z = \text{invnorm}(0.30) = -0.5244 = \frac{108 - \mu}{\sigma}$$

$$Z = \text{invnorm}(0.80) = 0.8416 = \frac{154 - \mu}{\sigma}$$

$$\begin{cases} -0.5244\sigma + \mu = 108 \\ 0.8416\sigma + \mu = 154 \end{cases}$$

$$-1.366\sigma = -46$$

$$\boxed{\sigma = 33.7 \text{ marks}}$$

$$0.8416(33.7) + \mu = 154$$

$$\boxed{\mu = 126 \text{ marks}}$$

Review

$\boxed{\#1}$ (a) $0.3 + \frac{1}{k} + \frac{2}{k} + 0.1 + 0.1 = 1$

$$\frac{3}{k} = 0.5$$

$$3 = 0.5k$$

$$k = 6$$

(b) $E(X) = 0.3(-2) + \frac{1}{6}(-1) + \frac{2}{6}(0) + 0.1(1) + 0.1(2)$

$$= -0.6 - \frac{1}{6} + 0.1 + 0.2$$

$$= -0.3 - \frac{1}{6}$$

$$= -\frac{3}{10} - \frac{1}{6} = -\frac{9}{30} - \frac{5}{30} = \frac{-14}{30}$$

$$= -\frac{7}{15}$$

#2

x	1	2	3	4	5
$P(X=x)$	$5c$	$8c$	$9c$	$8c$	$5c$
	$1/7$	$8/35$	$9/35$	$8/35$	$1/7$

(a) $5c + 8c + 9c + 8c + 5c = 1$

$$35c = 1$$

$$c = \frac{1}{35}$$

(b) $E(X) = \frac{1}{7}(1) + \frac{8}{35}(2) + \frac{9}{35}(3) + \frac{8}{35}(4) + \frac{1}{7}(5)$

$$= \frac{5}{35} + \frac{16}{35} + \frac{27}{35} + \frac{32}{35} + \frac{25}{35}$$

$$= \frac{105}{35} = 3$$

NOTE: $X = \frac{3}{8}$

#3 $P(3,3) + P(2,4) + P(4,2)$

$$= \frac{1}{8} \cdot \frac{1}{8} + \frac{1}{4} \cdot \frac{3}{8} + \frac{3}{8} \cdot \frac{1}{4}$$

$$= \frac{1}{64} + \frac{3}{32} + \frac{3}{32} = \frac{13}{64}$$

#4

	1	2	3	4
2	2	4	6	8
2	2	4	6	8
4	4	8	12	16
4	4	8	12	16

(a) 2, 4, 6, 8, 12, 16

(b) $P(2) = \frac{2}{16} = \frac{1}{8}$ $\quad P(6) = \frac{2}{16} = \frac{1}{8}$

$P(4) = \frac{4}{16} = \frac{1}{4}$ $\quad P(8) = \frac{4}{16} = \frac{1}{4}$

$P(12) = \frac{2}{16} = \frac{1}{8}$

$P(16) = \frac{2}{16} = \frac{1}{8}$

(c) $E(P) = \frac{1}{8}(2) + \frac{1}{4}(4) + \frac{1}{8}(6) + \frac{1}{4}(8) + \frac{1}{8}(12) + \frac{1}{8}(16)$

$= \frac{1}{4} + 1 + \frac{3}{4} + 2 + \frac{3}{2} + 2$

$= 7.5$

4d $P(10 \text{ or more}) = P(12) + P(16) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

$\frac{1}{4}(10) + \frac{3}{4}(5) = \frac{10}{4} + \frac{15}{4} = \frac{25}{4} = \underline{£6.25}$

per week on average

$\boxed{\text{After 10 weeks, £62.50}}$

$\boxed{\#5}$ $\binom{5}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 = 10 \cdot \frac{4}{243} = \frac{40}{243}$

$\boxed{\#6}$ $X \sim B(2, 0.1)$

$E(X) = np = 2(0.1) = \underline{0.2}$

$\boxed{\#7}$ $X \sim N(75, 5^2)$

a) $P(X < 65) = P(X > a)$ $\quad a = 75 + 2(5)$

$z = -2 \qquad z = +2 \qquad \boxed{a = 85}$

b) $P(65 < X < a) = \underline{0.954}$.

$z = -2 \qquad z = 2$

memorized value from normal distribution.

$P(z > 2) = \frac{1 - 0.954}{2} \quad \frac{0.023}{2}$

$= \underline{0.0115}$