15C #1 $X \sim B(4, \frac{1}{2})$ $\qquad (\frac{1}{2}+\frac{1}{2})^4$

a) $P(X=1) = \binom{4}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3$

$= 4\left(\frac{1}{2}\right)^4$

$= 4\left(\frac{1}{16}\right)$

$= \frac{1}{4}$

b) $P(X<1) = P(X=0) = \binom{4}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4$

$= \frac{1}{16}$

c) $P(X\le 1) = P(X=0) + P(X=1)$

$= \frac{1}{16} + \frac{1}{4} = \frac{5}{16}$

d) $P(X\ge 1) = 1 - P(X=0)$

$= 1 - \frac{1}{16} = \frac{15}{16}$

#2 a) binomialpdf$(6, 1/3, 2)$

b) $P(X < 2) = P(X \leq 1)$

$= \text{binomialcdf}(6, 1/3, 1)$

c) $P(X \leq 2) = \text{binomialcdf}(6, 1/3, 2)$

d) $P(X \geq 2) = 1 - P(X \leq 1)$

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Ex. $Y \sim B(10, \frac{1}{5})$ Calculator

a) $P(Y = 2) = 0.302$

b) $P(Y \leq 4) = 0.967$

c) $P(Y < 3) = P(Y \leq 2) = 0.678$

d) $P(Y > 2) = \underbrace{1 - P(Y \leq 2)} = 0.322$

$\boxed{\#3}$ Let $X$ = number of faulty components in a sample of 16.

15D

$X \sim B(16, 0.01)$

a) $P(X = 0) = 0.851$

b) $P(X = 3) = 0.000491$

c) $P(X \geq 2) = 1 - P(X \leq 1) = 0.0109$

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#4 $Y$ = Number of lines engaged

$Y \sim B(10, 0.25)$

a) $P(Y = 5) = 0.0584$

b) $P(Y \leq 7) = 0.9996$

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# Average Outcomes

Make a probability distribution table for $X$ = the number of ⚃ when 3 dice are rolled.

$X \sim B(3, \frac{1}{6})$ $\qquad \left(\frac{1}{6}+\frac{5}{6}\right)^3$

| $X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X=x)$ | $\frac{125}{216}$ | $\frac{75}{216}$ | $\frac{15}{216}$ | $\frac{1}{216}$ |

$$P(X=0) = \binom{3}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^3 = \frac{125}{216}$$

$$P(X=1) = \binom{3}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2 = \frac{75}{216}$$

$$P(X=2) = \binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1 = \frac{15}{216}$$

$$P(X=3) = \binom{3}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0 = \frac{1}{216}$$

The average number of ⚃s is represented by $E(X)$ or $\mu$ (mean)

↑ Expected value

$$E(X) = \frac{125}{216}(0) + \frac{75}{216}(1) + \frac{15}{216}(2) + \frac{1}{216}(3)$$

$$= 0 + \frac{75}{216} + \frac{30}{216} + \frac{3}{216} = \frac{108}{216}$$

$$= 0.5$$

$$\boxed{E(X) = np}$$ ← for a <u>binomial distribution</u>

$n$: number of trials

$p$: prob of success

---

A measure of dispersion (spread) — how spread out the data values are

<u>The variance</u> $(\sigma^2)$

For the <u>binomial distribution</u>:

$$\boxed{\sigma^2 = \text{Var}(X) = npq}$$

$q$: prob of failure

Ex. for $X \sim B(3, 1/6)$, $\text{Var}(X) = 3\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = \frac{15}{36}$

standard deviation $= \sigma = \frac{\sqrt{15}}{6}$

HW ① Roll 5 four-sided dice.

Let $X$ = the number of ◬ the are turned down

a) Make a probability distribution table.

| $X$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $P(X=x)$ | | | | | | |

b) Find the mean and variance of the distribution.

$\boxed{15D}$ # 5, 6 (calculator)

$\boxed{15C}$ # 3 (calculator)