#1 $(3x+2)^5$

$$(3x)^5 + \overset{5\,\cdot\,81}{\binom{5}{4}(3x)^4}(2) + \overset{10\,\cdot\,27\,\cdot\,4}{\binom{5}{3}(3x)^3(2)^2} + \overset{10\,\cdot\,9\,\cdot\,8}{\binom{5}{2}(3x)^2(2)^3}$$

$$+ \overset{5\,\cdot\,3\quad 16}{\binom{5}{1}(3x)(2)^4} + 2^5$$

$$= 243x^5 + 810x^4 + 1080x^3 + 720x^2$$

$$+ 240x + 32$$

#2 $(x^2-y)^4$

$$(x^2)^4 + \overset{4}{\binom{4}{3}}(x^2)^3(-y) + \overset{6}{\binom{4}{2}}(x^2)^2(-y)^2 + \binom{4}{1}(x^2)^1(-y)^3$$

$$+ (-y)^4$$

$$= \boxed{x^8 - 4x^6y + 6x^4y^2 - 4x^2y^3 + y^4}$$

The Binomial Distribution

Ex. Roll 4 dice. Count the number of ⚁

Find the probability of

a) no ⚁s $625/1296$

b) exactly 1 ⚁ $\frac{500}{1296}$

c) exactly 3 ⚁s $\frac{20}{1296}$

d) at least 1 ⚁

$1 - \frac{625}{1296} = \frac{671}{1296}$

• probability of a success is $p = \frac{1}{6}$

• probability of a failure is $q = \frac{5}{6}$

Expand $(p+q)^4$.

$$\left(\frac{1}{6}+\frac{5}{6}\right)^4 = \left(\frac{1}{6}\right)^4 + \overset{4}{\binom{4}{3}}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right) + \overset{6}{\binom{4}{2}}\left(\frac{1}{6}\right)^2\overset{25}{\left(\frac{5}{6}\right)^2}$$

$$+ \overset{4}{\binom{4}{1}}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^4$$

$$= \underbrace{\frac{1}{1296}}_{\text{prob. of 4 ⚁}} + \underbrace{\frac{20}{1296}}_{\text{prob of 3 ⚁}} + \underbrace{\frac{150}{1296}}_{\text{prob of 2 ⚁}} + \underbrace{\frac{500}{1296}}_{\text{prob of ⚁}} + \underbrace{\frac{625}{1296}}_{\text{prob of no ⚁s}}$$

Ex. Toss 40 coins. Count the number of heads.

a) $P(20 \text{ heads}) = \binom{40}{20}\left(\frac{1}{2}\right)^{20}\left(\frac{1}{2}\right)^{20} = 0.125$

b) $P(8 \text{ heads}) = \binom{40}{8}\left(\frac{1}{2}\right)^{8}\left(\frac{1}{2}\right)^{32} = 0.0000699$

c) $P(\text{no heads}) = 0.000000000000009$

$$\left(\frac{1}{2} + \frac{1}{2}\right)^{40}$$

A <u>Random Variable</u> is a quantity whose value depends on chance. (usually a capital letter)

Ex. (from today)

Let X = the number of ⚁s that show

$$P(X = 1) = \frac{500}{1296}$$

Ex. (from today)

Let Y = number of heads that show

$$P(Y > 20) = \frac{1 - \binom{40}{20}\left(\frac{1}{2}\right)^{20}\left(\frac{1}{2}\right)^{20}}{2} = 0.437$$

$$P(17 \le Y \le 23) = P(Y \le 23) - P(Y \le 16)$$

$$P(Y \ge 23) = 1 - P(Y \le 22)$$

$$= 0.215$$

$$X \sim B\left(6, \tfrac{1}{3}\right)$$

random variable

"has the distribution"

Binomial

n = number of trials

p = prob. of success

<u>HW</u> 15C #1, 2

no calc, calc

15D #3, 4

B(16, 0.01), B(10, 0.25)
