

# The Binomial Theorem

$$(x+2)^5 = \underbrace{(x+2)(x+2)(x+2)(x+2)(x+2)}$$

$$= x^5 + (2x^4)(\underline{5}) + (4x^3)(\underline{10})$$

$$+ (8x^2)(\underline{10}) + (16x)(\underline{5}) + 32$$

A combination:  $\binom{5}{3}$  read "5 choose 3"  
choosing "2" out of 3 of the binomials

## Formula

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}, \quad n! = n \cdot (n-1) \cdot (n-2) \cdots 1$$

$$\text{Ex. } \binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}}{\cancel{3} \cdot \cancel{2} \cdot 2} = 10$$

$$\text{Ex. } \binom{9}{4} = \frac{9!}{4! (9-4)!}$$

$$= \frac{9 \cdot \overset{2}{8} \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 126$$

$$\begin{array}{r} 5 \\ 18 \\ \times 7 \\ \hline 126 \end{array}$$

$$\text{Ex. } (2x - 3y)^4$$

$$= (2x)^4 + \binom{4}{3} (2x)^3 (-3y)^1 + \binom{4}{2} (2x)^2 (-3y)^2$$

$$+ \binom{4}{1} (2x)^1 (-3y)^3 + (-3y)^4$$

$$= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$

### Exercises

$$\textcircled{1} (3x + 2)^5$$

$$\textcircled{3} (4x + 2y)^6$$

$$\textcircled{2} (x^2 - y)^4$$

Math  $\rightarrow$  PROB  $\rightarrow$  nCr