

Ex.  $v(t) = t^2 - 9, t \geq 0$

$s(2) = 5 \leftarrow$  position at time  $t = 2$

$\uparrow$   
displacement (or distance) function

(a) Where is the particle when  $t = 6$ ?

$$s(t) = \int v(t) dt = \int (t^2 - 9) dt \rightarrow \left\{ \begin{array}{l} v(t) = \frac{ds}{dt} \\ a(t) = \frac{dv}{dt} \end{array} \right.$$

$$= \frac{1}{3}t^3 - 9t + C$$

$$s(2) = 5 = \frac{1}{3}(2)^3 - 9(2) + C$$

$$C = \frac{46}{3}$$

$$s(t) = \frac{1}{3}t^3 - 9t + \frac{46}{3}$$

$s(t)$  = displacement  
 $v(t)$  = velocity  
 $a(t)$  = acceleration

$$s(6) = \frac{1}{3}(6)^3 - 9(6) + \frac{46}{3} = \frac{100}{3}$$

(b) Is speed increasing when  $t = 6$ ?

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}[t^2 - 9] = 2t$$

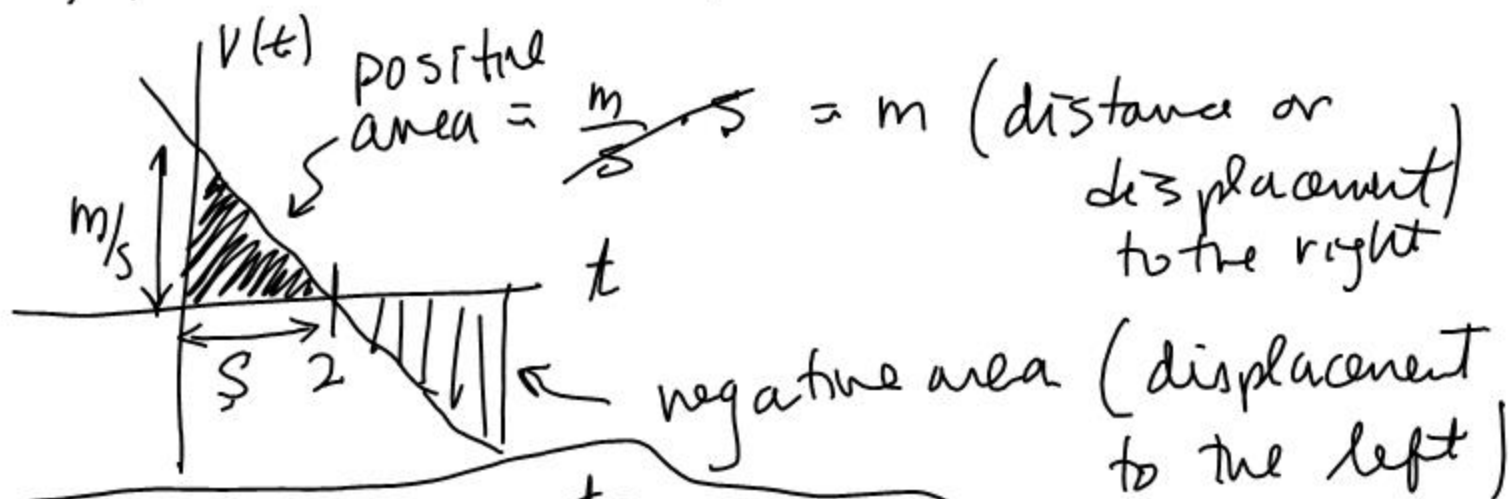
$$a(6) = 2(6) = 12 \leftarrow \text{positive, so the force is pulling to the right.}$$

$$v(6) = 6^2 - 9 = 27 \leftarrow \text{positive, so the particle is moving to the right}$$

So, the speed is increasing

travelled

(c) Find the distance in the 1st 6 seconds.



$$\text{displacement} = \int_{t_1}^{t_2} v(t) dt$$

$$\text{distance travelled} = \int_{t_1}^{t_2} |v(t)| dt$$

$$\text{distance travelled} = \int_0^6 |t^2 - 9| dt = 54$$

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Ex. The velocity of a particle is

$$v(t) = t^2 - 6t + 8 \quad t \geq 0$$

At time  $t = 0$ , the particle is at  $x = 4$ .

(a) Where is the particle at time  $t = 2$ .

(b) Is its speed increasing at time  $t = 2$ ?

(c) Find the distance travelled and the displacement in first 8 seconds.

$$\begin{aligned} \text{(a) } s(t) &= \int (t^2 - 6t + 8) dt \\ &= \frac{1}{3}t^3 - 3t^2 + 8t + C \end{aligned}$$

$$s'(0) = C = 4$$

$$s(t) = \frac{1}{3}t^3 - 3t^2 + 8t + 4$$

$$s'(2) = \frac{1}{3}(2)^3 - 3(2)^2 + 8(2) + 4 = \frac{32}{3}$$

$$(b) \quad v(2) = 2^2 - 6(2) + 8 = 0$$

The particle is at rest at time  $t = 2$ .

$$(c) \quad \text{distance travelled} = \int_0^8 |t^2 - 6t + 8| dt \\ = \frac{136}{3}$$

$$(d) \quad \text{displacement} = \int_0^8 (t^2 - 6t + 8) dt = \frac{128}{3}$$