$\boxed{14E}$

\#1. $\int (2\cos x + 3\sin x)\,dx$

$= 2\sin x - 3\cos x + C$

\#2. $\int x^2\,dx + 3\int \cos\left(\frac{1}{3}x\right)\frac{1}{3}dx$

$\boxed{\frac{1}{3}x^3 + 3\sin\left(\frac{1}{3}x\right) + C}$

$u = \frac{1}{3}x$ $\quad 3\int \cos u\,du$

$du = \frac{1}{3}dx$

\#3 $\int \pi \sin(\pi x)\,dx = \int \sin u\,du$

$u = \pi x$

$du = \pi\,dx$

$= -\cos(\pi x) + C$

\#4. $\frac{1}{2}\int 2\sin(2x+3)\,dx = \frac{1}{2}\int \sin u\,du$

$u = 2x+3$

$du = 2\,dx$

$= -\frac{1}{2}\cos(2x+3) + C$

#5. $\int 20x^3 \cos(5x^4)\, dx$

$u = 5x^4$

$du = 20x^3\, dx$

$= \int \cos u \, du = \sin(5x^4) + C$

#6 $\frac{1}{4}\int 4(2x-1) \cos(4x^2 - 4x)\, dx$

$u = 4x^2 - 4x$

$du = (8x - 4)\, dx$

$= \frac{1}{4}\int \cos u \, du = \frac{1}{4} \sin(4x^2 - 4x) + C$

composite

#7. $\int \frac{e^{\tan 3x}}{\cos^2 3x}\, dx$

$u = \tan 3x$

$du = -\ln(\cos 3x) \cdot 3\, dx$

#8. $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx = \int \cos u\,du$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \sin(\ln x) + C$

#9. $\displaystyle\int \cos x \sin^2 x\,dx = \int u^2\,du$

$u = \sin x$

$du = \cos x\,dx$

$= \frac{1}{3}u^3 + C$

$= \frac{1}{3}\sin^3 x + C$

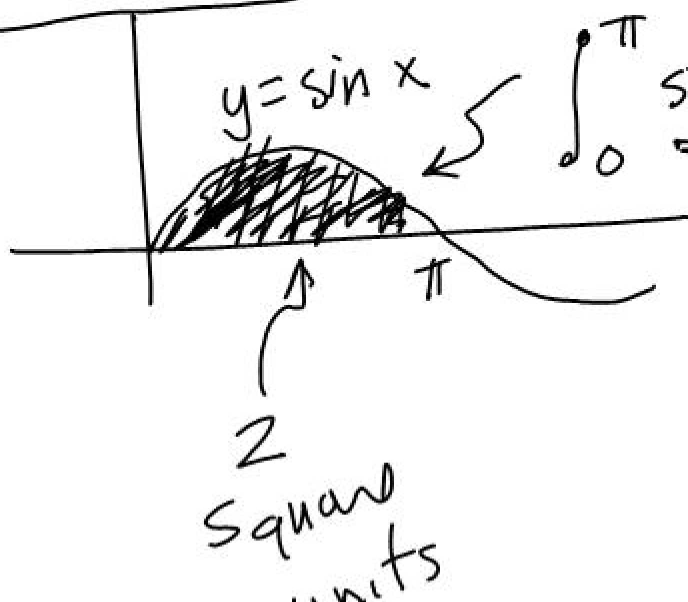
$\sin^2 x = (\sin x)^2$

The Fundamental theorem of Calculus (FTC)

$$\int_a^b \underline{f'}(x)\,dx = f(b) - f(a)$$

OR $$\int_a^b f(x)\,dx = F(b) - F(a),$$

$$F'(x) = f(x)$$

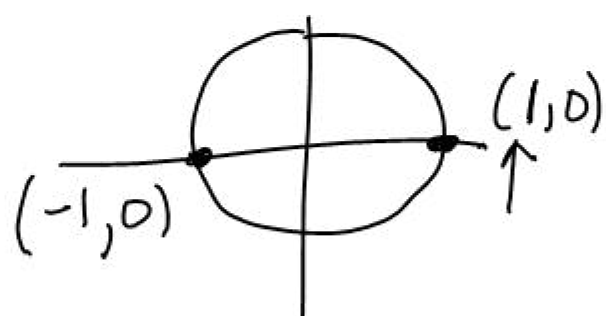


$$\int_0^\pi \underline{\sin x}\,dx = -\cos(\pi) + \cos 0$$

$$= -(-1) + 1 = 2$$

$f'(x) = \sin x$

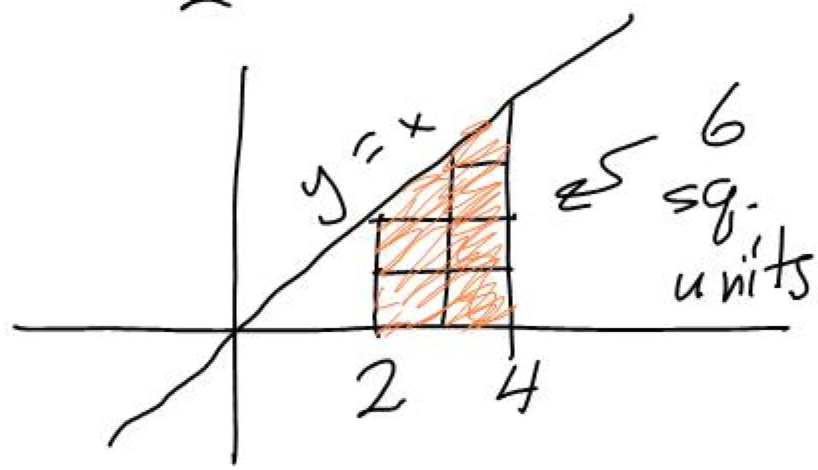
$f(x) = -\cos x$



$F(x)$ is the antiderivative of $f(x)$

$$\int f(x)\,dx = F(x)$$

Definite Integral: $\int_2^4 x\,dx = 6$



Antiderivative

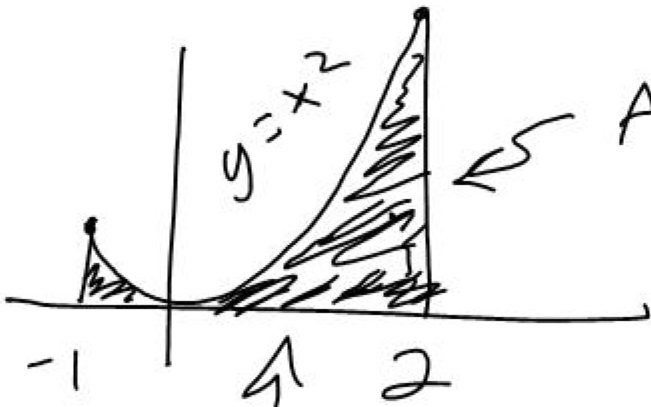
<u>or</u>

Indefinite integral

$$\int x\,dx = \frac{1}{2}x^2 + c$$

<u>FTC</u>:

$$\int_2^4 x\,dx = \frac{1}{2}(4)^2 - \frac{1}{2}(2)^2$$
$$= 8 - 2$$
$$= 6$$



Area $= \int_{-1}^{2} x^2 \, dx$

$= \left[\frac{1}{3} x^3 \right]_{-1}^{2}$

antiderivative of x^2

$= \frac{1}{3}(2)^3 - \frac{1}{3}(-1)^3$

$= \frac{8}{3} - \frac{-1}{3} = 3$

Ex. Use the FTC: $\int_0^1 (x^4 - x^3)\, dx$

$= \left[\frac{1}{5} x^5 - \frac{1}{4} x^4 \right]_0^1 = \left(\frac{1}{5}(1)^5 - \frac{1}{4}(1)^4 \right) - 0$

$= \frac{4}{20} - \frac{5}{20} = \frac{-1}{20}$

Exercises Use the FTC

(1) $\int_2^5 x^2\,dx$

(2) $\int_0^{\pi} \cos x\,dx$

(3) $\int_0^{\ln 2} e^x\,dx$

(4) $\int_1^{e^2} \frac{1}{x}\,dx$

(5) $\int_{\pi/3}^{\pi/2} \sin x\,dx$

(6) $\int_0^4 \sqrt{x}\,dx$