u-substitution

A way to get an anti derivative of a composite function

Ex. $\frac{1}{3}\int \overbrace{(3x+4)^6}^{\text{composite}} 3\,dx = \frac{1}{3}\int u^6\,du$

inner function $\Rightarrow u = 3x+4$ $\quad = \frac{1}{3}\cdot\frac{1}{7}u^7 + C$

$\frac{du}{dx} = 3$, $du = 3\,dx$ $\quad = \frac{1}{21}(3x+4)^7 + C$

$y = x^2$

$\frac{dy}{dx} = 2x$

The Power Rule for Antiderivs.

$$\boxed{\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1}$$

check $\frac{d}{dx}\left[\frac{1}{21}(3x+4)^7 + C\right]$

$= \frac{1}{\cancel{3}}\frac{\cancel{7}}{\cancel{21}}(3x+4)^6 \cdot \cancel{3} = (3x+4)^6$

Ex. $\frac{1}{12}\int 12x^2 \underbrace{\sqrt{4x^3+1}}_{\text{composite function}}\, dx = \frac{1}{12}\int \sqrt{u}\, du$

inner function $\rightarrow$ $u = 4x^3+1$

$du = 12x^2\, dx$

$$= \frac{1}{12}\int u^{1/2}\, du$$

$$= \frac{1}{\cancel{12}_{6}} \cdot \frac{\cancel{2}}{3} u^{3/2} + C$$

$$= \boxed{\frac{1}{18}\left(4x^3+1\right)^{\frac{3}{2}} + C}$$

check

$\frac{d}{dx}\left[\frac{1}{18}\left(4x^3+1\right)^{3/2}\right]$ ← use the chain rule

$$= \frac{3}{\cancel{2}} \cdot \frac{1}{\cancel{18}_{6}}\left(4x^3+1\right)^{1/2} \cdot \cancel{12}\, x^2$$

$$= x^2\sqrt{4x^3+1}$$

Note u-sub. doesn't always work

$$\text{Ex. } \cancel{\frac{1}{12x^2}\int 12x^2\sqrt{4x^3+1}\,dx}$$

$$\cancel{u = 4x^3+1}$$

$$\cancel{du = 12x^2\,dx}$$

$$\text{Ex. } \frac{1}{2}\int 2x\cdot \overbrace{e^{x^2}}^{\text{composite}}\,dx = \frac{1}{2}\int e^u\,du$$

$$u = x^2$$

$$du = 2x\,dx$$

$$= \frac{1}{2}e^u + C$$

$$= \boxed{\frac{1}{2}e^{x^2} + C}$$

Ex. $\frac{1}{3}\int \frac{1}{3x+2} \underline{3\, dx} = \frac{1}{3}\int \frac{1}{u}\, du$

denom. $\rightarrow$ $u = 3x+2$

$du = \underline{3\, dx}$

$= \frac{1}{3} \ln u + C$

$= \frac{1}{3} \ln(3x+2) + C$

or

$\frac{1}{3} \ln |3x+2| + C$

$$\boxed{\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}}$$

Ex. $-\int \frac{-\sin x}{\cos x}\, dx = -\int \frac{1}{u}\, du$

denom $\rightarrow$ $u = \cos x$

$du = -\sin x\, dx$

$= -\ln u + C$

$= -\ln(\cos x) + C$

$$\int \tan x \, dx = -\ln(\cos x) + C$$

Note $\int \tan x \, dx = \ln|\sec x| + C$

HW $\boxed{14E}$ #1–6, 8, 9

$u = \ln x$