SL Test Review

1. $f(x) = 3x + 2 \Rightarrow f'(x) = 3$

2. $f(x) = 4x^2 - 5x + 3$

   $f'(x) = 8x - 5$

3. $f(x) = (3x-2)^4$

   $f'(x) = 4(3x-2)^3 \cdot 3 = 12(3x-2)^3$

4. $f(x) = e^{5x+6} \Rightarrow f'(x) = 5 \cdot e^{5x+6}$

5. $f(x) = \dfrac{2x+4}{3x+1}$

$$f'(x) = \frac{(3x+1)(2) - (2x+4)(3)}{(3x+1)^2}$$

$$= \frac{6x + 2 - 6x - 12}{(3x+1)^2}$$

$$= \frac{-10}{(3x+1)^2}$$

6 $f(x) = x^3 \cdot e^{5x}$

$$f'(x) = x^3 \cdot 5e^{5x} + e^{5x} \cdot 3x^2$$

$$= x^2 e^{5x}(5x+3)$$

7 $f(x) = \frac{1}{x^3} = x^{-3}$

$$f'(x) = -3x^{-4} = \frac{-3}{x^4}$$

8 $f(x) = \sqrt{4x+3} = (4x+3)^{1/2}$

$$f'(x) = \frac{1}{2}(4x+3)^{-1/2} \cdot 4 = \frac{2}{\sqrt{4x+3}}$$

9 $f(x) = \frac{e^x}{x^3} \Rightarrow f'(x) = \frac{x^3 \cdot e^x - e^x \cdot 3x^2}{x^6}$

$$= \frac{x^2 e^x (x-3)}{x^6} = \frac{e^x(x-3)}{x^4}$$

10) $f(x) = x^3 - 3x^2 + 2$

$$f'(x) = 3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$x = 0 \text{ or } x = 2$$

$f'(x)$: $\quad + \quad | \quad - \quad | \quad +$
$\qquad\qquad\quad\ \ 0 \qquad\quad\ 2$

f is incr. on $(-\infty, 0) \cup (2, \infty)$
f has a min. at $x=2$ and a max at $x=0$.

$$f''(x) = 6x - 6 = 0$$

$$x = 1$$

$f''(x)$: $\quad - \quad | \quad +$
$\qquad\qquad\quad\ \ 1$

f is concave up on $(1, \infty)$.
f has a flexpt. at $x=1$

11 $f(x) = xe^x$

$$f'(x) = xe^x + e^x = e^x(x+1) = 0$$

$$x = -1$$

$f'(x)$: $-$ $+$

-1

$$\boxed{\begin{array}{l} f \text{ is incr. on } (-1, \infty) \\ f \text{ has a min. at } x = -1 \end{array}}$$

$$f''(x) = xe^x + e^x + e^x$$

$$= e^x(x+2) = 0$$

$$x = -2$$

$f''(x)$: $-$ $+$

-2

$$\boxed{\begin{array}{l} f \text{ is concave up on } (-2, \infty), \\ f \text{ has a flex pt. at } x = -2. \end{array}}$$

(12) $f(x) = \dfrac{1}{x^2+1} = (x^2+1)^{-1}$

$$f'(x) = -(x^2+1)^{-2} \cdot 2x = \frac{-2x}{(x^2+1)^2}$$

$$f'(x):\ \begin{array}{ccc} + & & - \\ \hline & 0 & \end{array}$$

$$\boxed{\begin{array}{l} f \text{ is incr. on } (-\infty, 0). \\ f \text{ has a max. at } x = 0. \end{array}}$$

$$f''(x) = \frac{(x^2+1)^2 \cdot (-2) - (-2x)\, 2(x^2+1)(2x)}{(x^2+1)^4}$$

$$= \frac{(x^2+1)(-2)\left[x^2+1-4x^2\right]}{(x^2+1)^4}$$

$$= \frac{-2(1-3x^2)}{(x^2+1)^3} = 0 \qquad x = \pm\frac{1}{\sqrt{3}}$$

$$f''(x):\ \begin{array}{ccccc} + & & - & & + \\ \hline & -\frac{1}{\sqrt{3}} & & \frac{1}{\sqrt{3}} & \end{array}$$

$$\boxed{\begin{array}{l} f \text{ is concave up on } \left(-\infty, -\frac{1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}}, \infty\right) \\ f \text{ has a flex pt. at } x = \pm\frac{1}{\sqrt{3}} \end{array}}$$

(13) $A = xy$; $5x + 2y = 4000$

$2y = 4000 - 5x$

$y = 2000 - \frac{5}{2}x$

$A = x\left(2000 - \frac{5}{2}x\right)$

$A = 2000x - \frac{5}{2}x^2$

$A' = 2000 - 5x = 0$

$\boxed{x = 400\text{ ft}}$

$y = 2000 - \frac{5}{\cancel{2}}(\cancel{400})$ 200

$\boxed{y = 1000\text{ ft}}$

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(14) F = amount of fence

$F = 5x + 2y$; $A = x \cdot y = 10000$

$y = \frac{10000}{x}$

$F = 5x + 2\left(\frac{10000}{x}\right)$

$F = 5x + \frac{20000}{x} = 5x + 20000x^{-1}$

$F'(x) = 5 - 20000x^{-2} = 5 - \frac{20000}{x^2} = 0$

$5x^2 - 20000 = 0$

$x^2 = 4000 \Rightarrow x = \sqrt{4000} = \sqrt{400 \cdot 10} = \underline{20\sqrt{10}\text{ ft}}$

$$y = \frac{1000\cancel{0}}{2\cancel{0}\sqrt{10}} = \frac{500}{\sqrt{10}} \text{ ft.}$$

\#15 $f(x) = \sqrt{x} = x^{1/2}$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$f(9) = \sqrt{9} = 3$$

$$f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{2\cdot 3} = \frac{1}{6} = \text{slope}$$

$$\boxed{\text{Tangent line : } y - 3 = \frac{1}{6}(x-9)}$$

16 $f(x) = x^3 - x$

$$f'(x) = 3x^2 - 1$$

$$f(1) = 1^3 - 1 = 0$$

$$f'(1) = 3(1)^2 - 1 = 2 = \text{slope}$$

tangent line : $y - 0 = 2(x-1)$

or $y = 2x - 2$