$\boxed{7S}$ #8 $\quad f''(x)$: $\quad + \;\; \big|_{-2} \;\; - \;\; \big|_{4} \;\; +$

Concave up $(-\infty, -2) \cup (4, \infty)$

flex points: $x = -2$, $x = 4$

$\boxed{7T}$ #6 $\quad f(x) = \dfrac{x^2 - 1}{x^2 + 1}$

$$f'(x) = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$$

$$= \frac{2x(\cancel{x^2}+1 - \cancel{x^2}+1)}{(x^2+1)^2}$$

$$= \frac{4x}{(x^2+1)^2}$$

$$f''(x) = \frac{(x^2+1)^2 \cdot 4 - 4x \cdot 2(x^2+1)^1(2x)}{(x^2+1)^4}$$

$$= \frac{4(x^2+1)[x^2+1 - 4x^2]}{(x^2+1)^4}$$

$$f'(x) = \frac{4x}{(x^2+1)^2}$$

$$f''(x) = \frac{4(x^2+1)(1-3x^2)}{(x^2+1)^4}$$

$1-3x^2=0$

$3x^2=1$

$x^2=\frac{1}{3}$

$x=\pm\frac{1}{\sqrt{3}}$

$f'(x)$: $-$ | 0 | $+$

$f''(x)$: $-$ | $-\frac{1}{\sqrt{3}}$ | $+$ | $\frac{1}{\sqrt{3}}$ | $-$

x-int ± 1

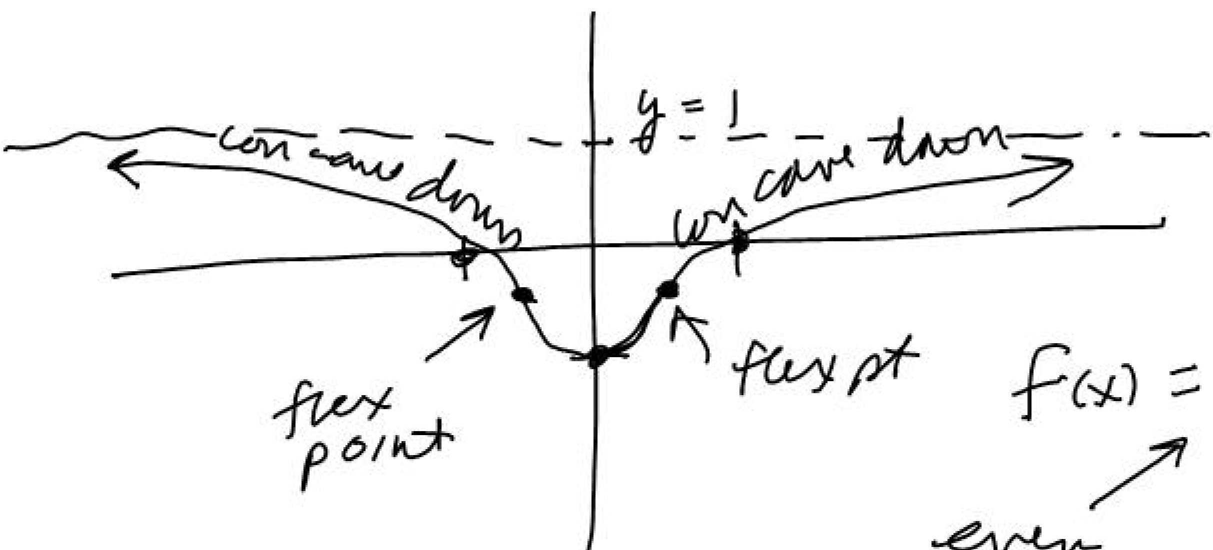
VA NONE

$x^2-1=0$

y-int -1

HA $y=1$

$(x-1)(x+1)=0$



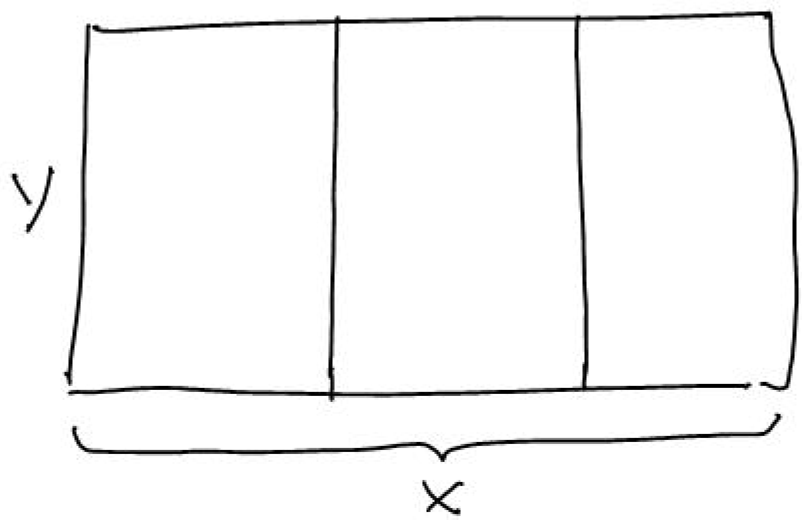
$$f(x)=\frac{x^2-1}{x^2+1}$$

even function

Optimization

Ex. A rancher has 1000 ft of fence material to build a corral as shown:

Find the dimensions for the largest corral.



Area function: $A = xy$ ← find where this has a maximum

$$2x + 4y = 1000$$

$$4y = 1000 - 2x$$

$$y = 250 - \frac{1}{2}x$$

$$A = x\left(250 - \frac{1}{2}x\right) = 250x - \frac{1}{2}x^2$$

$$A = 250x - \frac{1}{2}x^2$$

$$A'(x) = 250 - x = 0$$

$$x = \underline{250 \text{ ft}}$$

$$y = 250 - \frac{1}{2}(250)$$

$$y = \underline{125 \text{ ft}}$$

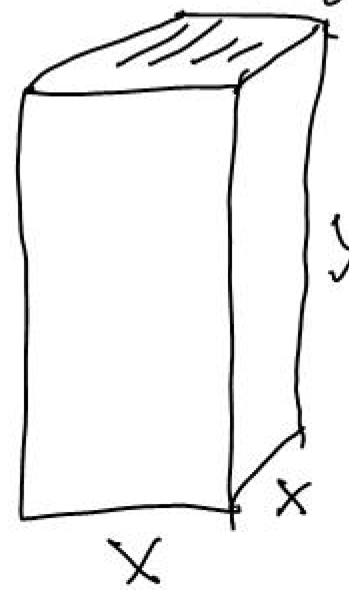
Find the x-value of the vertex $\cap$ $\cup$
for $f(x) = ax^2 + bx + c$, $a \neq 0$

$$f'(x) = 2ax + b = 0$$

$$2ax = -b$$

$$\boxed{x = \frac{-b}{2a}}$$ $\cap$ $\underline{\cup}$

Ex. An open-top, rectangular box with a square bottom is to have 4 ft^3 of volume. Find the dimensions of a box with minimum surface area.



Surface Area:

$$A = x^2 + 4xy$$

$$V = x^2 y = 4$$

$$y = \frac{4}{x^2}$$

$$A = x^2 + 4x\left(\frac{4}{x^2}\right)$$

$$A = x^2 + \frac{16}{x}$$ ← find the minimum

$$A' = 2x + \frac{0 - 16}{x^2} = 2x - \frac{16}{x^2} = 0$$

$$2x^3 - 16 = 0$$

$$x^3 = 8$$

$$x = \underline{2 \text{ ft.}}$$

$$\underline{y = 1 \text{ ft}}$$

Ex. The cost of manufacturing x items is given by

$$C(x) = x^3 - 3x^2 + 4 \quad (x \text{ in hundreds})$$

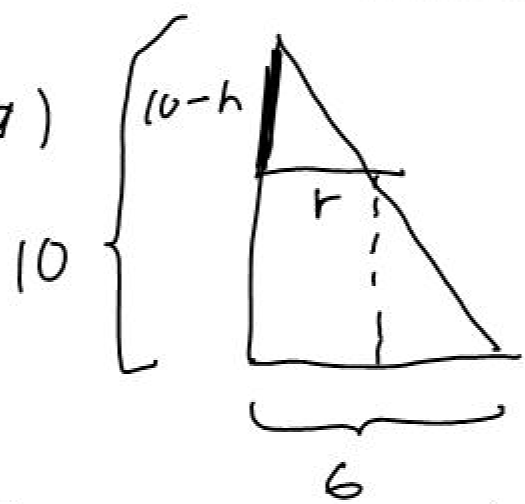
Find the minimum cost.

$$C'(x) = 3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$\cancel{x = 0} \quad \underline{\text{or}} \quad \underline{\underline{x = 2}}$$

$\boxed{TY}$ $\underline{\#\ 4}$ (a)



$$\frac{\cancel{r}}{\cancel{6}} = \left(\frac{10-h}{\cancel{10}_{\,5}}\right) \times 3$$

$$\boxed{r = \frac{30-3h}{5}}$$

(b) Volume of the cylinder: $V_c = \pi r^2 h$

$$V_c = \pi\left(\frac{30-3h}{5}\right)^2 \cdot h$$ use FOIL

$$V_c = \pi\left(\frac{900-120h+9h^2}{25}\right)h$$

$$V_c = \pi\left(\frac{900h-120h^2+9h^3}{25}\right)$$

HW $\boxed{\begin{array}{l} 7X \# 3 \\ 7Y \# 1,2,3 \end{array}}$

To be continued!