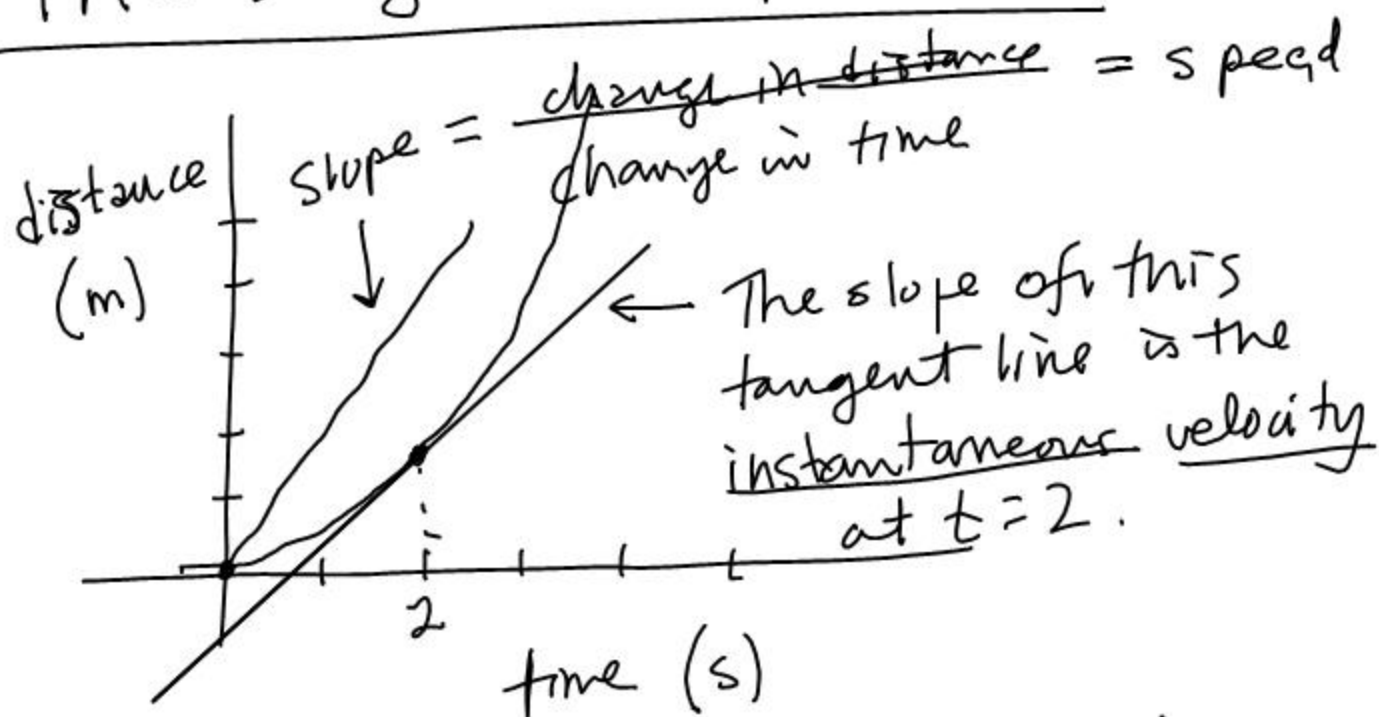
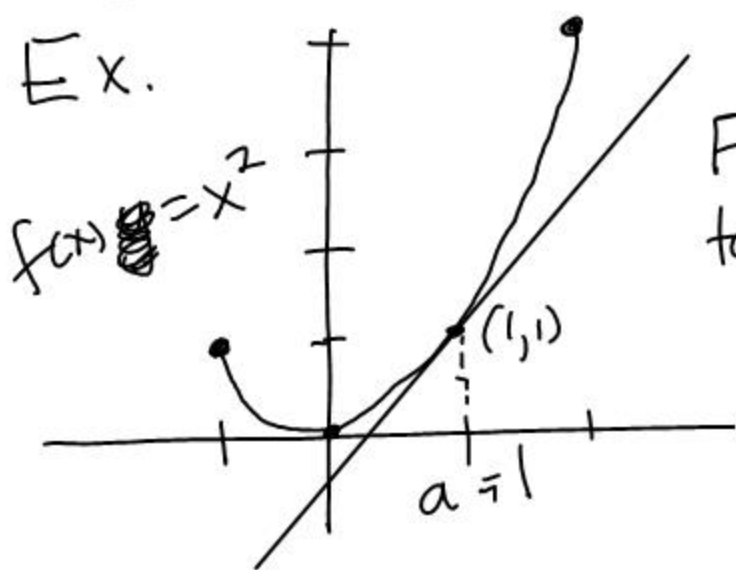
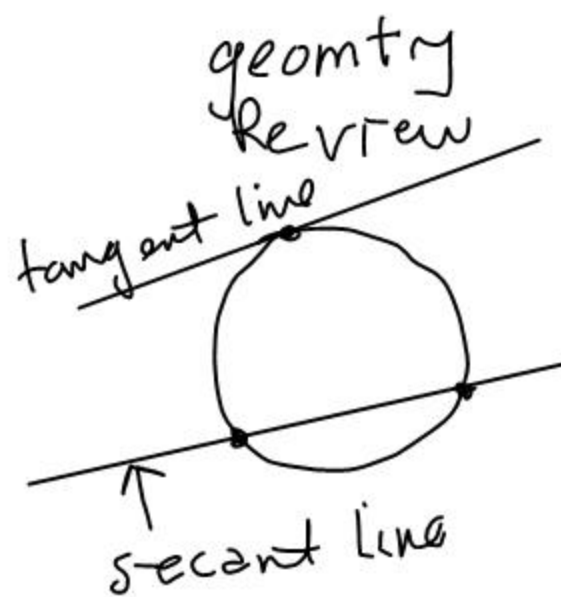


The tangent line problem

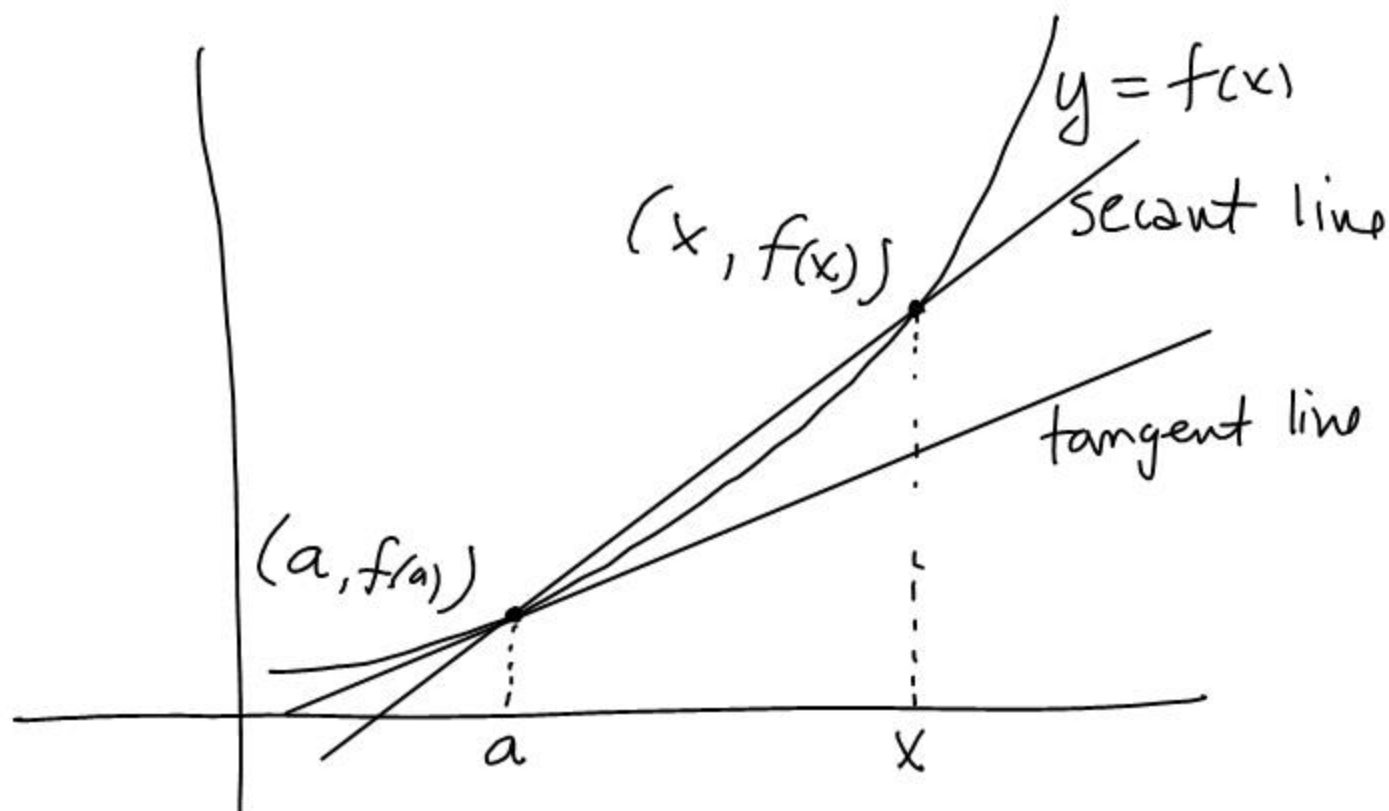


How do we get the slope of a tangent line?



Find the slope of the tangent line at $(1, 1)$ $f(1)$

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

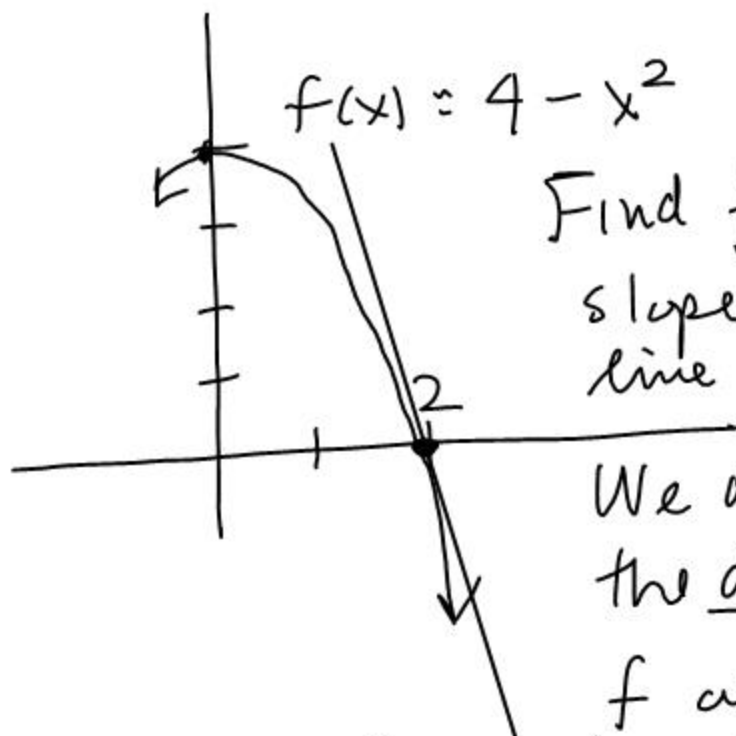


secant line slope is $\frac{f(x) - f(a)}{x - a}$

The tangent line slope is $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x+1)(x-1)}{x-1} = 2$$

Ex.



Find $f'(2)$, the slope of the tangent line at $x=2$.

We also call this the derivative of f at $x=2$.

$$f'(2) = \lim_{x \rightarrow 2} \frac{(4 - x^2) - (4 - 2^2)}{x - 2}$$

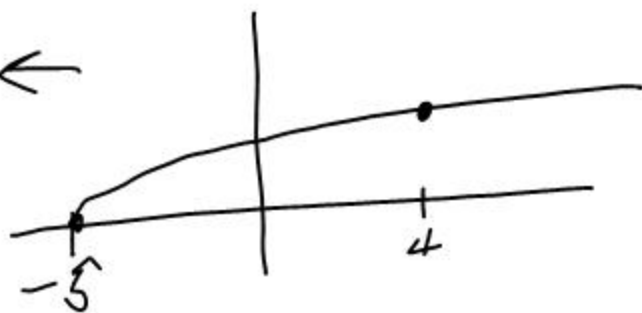
$$= \lim_{x \rightarrow 2} \frac{4 - x^2}{x - 2}$$

$$= \lim_{x \rightarrow 2} \frac{(2 - x)(2 + x)}{x - 2} = -1(2 + 2)$$

$$\lim_{x \rightarrow 2} \frac{-(x^2 - 4)}{x - 2} = \lim_{x \rightarrow 2} \frac{-(x + 2)(x - 2)}{x - 2} = -4$$

Ex. $f(x) = \sqrt{x+5}$ ←

Find $f'(4)$



$$f'(4) = \lim_{x \rightarrow 4} \frac{\sqrt{x+5} - \sqrt{4+5}}{x-4}$$

$$= \lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x-4} \cdot \frac{\sqrt{x+5} + 3}{\sqrt{x+5} + 3}$$

$$= \lim_{x \rightarrow 4} \frac{\cancel{x-4} \cdot 9}{(\cancel{x-4}) (\sqrt{x+5} + 3)} = \frac{1}{6}$$