The Binomial Theorem

Ex. $(3x+2)^5 = (3x+2)(3x+2)(3x+2)(3x+2)(3x+2)$

$$= \underline{(3x)^5} + \underbrace{\overset{5}{\binom{5}{1}}\overset{81}{(3x)^4}\overset{2}{(2)^1}} + \underbrace{\overset{10}{\binom{5}{2}}\overset{27}{(3x)^3}\overset{4}{(2)^2}}$$

$$+ \underbrace{\overset{10}{\binom{5}{3}}\overset{9}{(3x)^2}\overset{8}{(2)^3}} + \overset{5}{\binom{5}{4}}\overset{3}{(3x)^1}\overset{16}{(2)^4} + 2^5$$

$$= 243x^5 + 810x^4 + 1080x^3 \cdot 720x^2 + 240x + 32$$

Combinations

$$\binom{5}{2} = \frac{5!}{2!\,(5-2)!} = \frac{5\cdot\cancel{4}^{2}\cdot\cancel{3}\cdot\cancel{2}}{\cancel{2}\cdot\cancel{3}\cdot\cancel{2}} = 10$$

$$\boxed{\binom{n}{r} = \frac{n!}{r!\,(n-r)!}}$$

Ex. $(x^2 - 2y)^4$

$$= (x^2)^4 + \overset{4}{\binom{4}{1}}(x^2)^3\overset{-2}{(-2y)^1} + \overset{6}{\binom{4}{2}}(x^2)^2\overset{4}{(-2y)^2}$$

$$+ \overset{4}{\binom{4}{3}}(x^2)^1\overset{-8}{(-2y)^3} + \overset{16}{(-2y)^4}$$

$$= \underline{x^8 - 8x^6y + 24x^4y^2 - 32x^2y^3 + 16y^4}$$

$$\binom{4}{2} = \frac{4!}{2!\,2!} = \frac{\overset{2}{\cancel{4}}\cdot 3\cdot \cancel{2}}{\cancel{2}\cdot \cancel{2}} = 6$$

The Binomial Distribution

Ex. Roll 4 dice. Count the number of ⚁

Find the probability of

a) no ⚁ 625/1296

b) exactly 1 ⚁ 500/1296

c) exactly 3 ⚁ 20/1296

d) at least 1 ⚁

$1 - P(\text{no ⚁}) = 1 - \frac{625}{1296} \to \frac{671}{1296} \approx 51.8\%$

- probability of a success: $p = \frac{1}{6}$
- probability of a failure: $q = \frac{5}{6}$
- n = number of trials

Expand $(p+q)^n$

$$\left(\frac{1}{6}+\frac{5}{6}\right)^4 = \left(\frac{1}{6}\right)^4 + \underbrace{\binom{4}{1}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1}_{}$$

$$+ \overset{6 \cdot 25}{\underbrace{\binom{4}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2}} + \overset{4 \cdot 125}{\underbrace{\binom{4}{3}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3}} + \left(\frac{5}{6}\right)^4$$

$$= \frac{1}{1296} + \frac{20}{1296} + \frac{150}{1296} + \frac{500}{1296} + \frac{625}{1296}$$

A <u>Random Variable</u> is a quantity whose value depends on chance. (usually we use a capital letter).

Ex. Let X = the number of ⚁s that show when 4 dice are rolled.

$$P(X=1) = \frac{500}{1296}$$

$$P(X \geq 1) = \frac{671}{1296}$$

Ex. Toss 40 coins. $\left(\frac{1}{2}+\frac{1}{2}\right)^{40}$

Let Y = the number of heads

a) $P(Y=20) = \binom{40}{20}\left(\frac{1}{2}\right)^{20}\left(\frac{1}{2}\right)^{20}$

$= 0.125$

b) $P(Y=15) = \binom{40}{15}\left(\frac{1}{2}\right)^{15}\left(\frac{1}{2}\right)^{25}$

$.0366$

90%
80%
50%
25%
40%
45%
37%

7%
4.2%
3%
6.5%

c) $P(Y \geq 15) = 1 - \underbrace{P(Y \leq 14)}_{\text{binomialcdf}(\underset{n}{40},\ \underset{p}{0.5},\ 14)}$

$= 0.959$

d) $P(18 \leq Y \leq 22) = P(Y \leq 22) - P(Y \leq 17)$

$X \sim \underset{\text{Binomial distri}}{B}\left(\underset{\text{number trials}}{6},\ \underset{\text{prob of success}}{\tfrac{1}{3}}\right)$

HW 15C #1, 2

15D #3, 4 (calc)
