Probability Distributions

Discrete

x	0	1	2
P(x)	1/2	1/3	

↑ find the missing probability

- A special discrete distribution is the binomial distribution

parameters: p (prob. of success)

n (number of trials)

$\mu = np$

$\sigma^2 = npq$, $q = 1 - p$

Continuous

- A special cont. distr. is the normal distribution.

parmeters: μ, σ^2

If $X \sim B(n, p)$,

$$P(X = x) = \binom{n}{x} (p)^x (q)^{n-x}$$

x ↑ number of successes

Review

Ex. X is a discrete random variable

x	0	1	2	3
$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(a) Find the mean and variance of X.

$$\mu = \frac{1}{8}(0) + \frac{3}{8}(1) + \frac{3}{8}(2) + \frac{1}{8}(3)$$

$$= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = 1\frac{1}{2}$$

Note: $\mu = E(X)$, the expected value of X

$$E(X^2) = \frac{1}{8}(0^2) + \frac{3}{8}(1^2) + \frac{3}{8}(2^2) + \frac{1}{8}(3^2)$$

$$= \frac{3}{8} + \frac{12}{8} + \frac{9}{8}$$

$$= 3$$

$$\boxed{\sigma^2 = E(X^2) - [E(X)]^2}$$

$$= 3 - \left(\frac{3}{2}\right)^2$$

$$= \frac{12}{4} - \frac{9}{4} = \boxed{\frac{3}{4}} \text{ variance of } X$$

standard deviation of X: $\sigma = \sqrt{\frac{3}{4}}$

$$= \frac{\sqrt{3}}{2}$$

(b) $P(X \geq 1) = \frac{7}{8}$

(c) $P(X < 2) = \frac{1}{2}$

Ex. Roll 6 dice. X = number of ⚄

(a) $P(X=3) = \binom{6}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^3 = \frac{2500}{6^6}$

$$\binom{6}{3} = \frac{6!}{3!\,(6-3)!} = \frac{6\cdot 5\cdot 4\cdot 3!}{3\cdot 2\cdot 3!} = 20$$

$$\boxed{\binom{n}{r} = \frac{n!}{r!\,(n-r)!}}$$

$$\begin{array}{r} 125 \\ \times\ 20 \\ \hline 2500 \end{array}$$

(b) $P(X=3)$ use a calculator

binomialpdf (6, 1/6, 3)