

① Toss 5 coins. $X =$ number of (H)

x	0	1	2	3	4	5
$P(X=x)$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

$$P(X=0) = \binom{5}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

\uparrow \uparrow
 p q

$$P(X=1) = \binom{5}{1} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 = 5 \left(\frac{1}{2}\right) \left(\frac{1}{16}\right) = \frac{5}{32}$$

$$P(X=2) = \binom{5}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 = 10 \left(\frac{1}{4}\right) \left(\frac{1}{8}\right) = \frac{10}{32}$$

$$P(X=3) = \binom{5}{3} \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^2$$

$$\begin{array}{ccc} 10 & \frac{1}{8} & \frac{1}{4} \\ & \underbrace{\hspace{2em}} & \\ & \frac{1}{32} & \\ \frac{10}{32} & & \end{array}$$

$$P(X=4) = \binom{5}{4} \cdot \left(\frac{1}{2}\right)^4 \cdot \left(\frac{1}{2}\right)^1$$

$$5 \cdot \frac{1}{16} \cdot \frac{1}{2} = \frac{5}{32}$$

$$P(X=5) = \binom{5}{5} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 = \frac{1}{32}$$

n trials

$$P(X=x) = \binom{n}{x} (p)^x (q)^{n-x}$$

\uparrow prob. of success \uparrow $q = 1 - p$

$$(b) E(X) = np = 5 \cdot \frac{1}{2} = 2.5$$

$$(c) \sigma^2 = npq = 5 \cdot \frac{1}{2} \cdot \frac{1}{2} = 1.25$$

(1/2)

Draw 3 cards from a deck
with replacement.

X = number of ♥

$$\text{Find } P(X=2) = \binom{3}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^1$$

$$= 3 \cdot \frac{1}{16} \cdot \frac{3}{4}$$

$$= \frac{9}{64}$$

(1) $\frac{3}{4}$

A bent coin lands (H) 45% of the time. Toss 4 coins.

X = number of (T)

$$\begin{aligned} \text{Find } P(X=2) &= \binom{4}{2} (0.55)^2 (0.45)^2 \\ &= \underline{\underline{0.368}} \end{aligned}$$

(2)

Y	0	1	2
$P(Y=y)$	$\frac{1}{2}$	$\frac{1}{10}$	$4x$

(a) Find x : $\frac{1}{2} + x + 4x = 1$

(b)

$$\begin{aligned} E(Y) &= \frac{1}{2}(0) + \frac{1}{10}(1) + \frac{2}{5}(2) \\ &= \frac{9}{10} \end{aligned}$$

$$5x = \frac{1}{2}$$

$$x = \frac{1}{10}$$