

# Probability Distributions

A random variable represents the outcome of an experiment. (capital letters are used).

Ex. Discrete random variable

$X$  = number of heads when 5 coins are tossed

Ex. Continuous random variable

$Y$  = height of a randomly selected freshman.

Roll 3 dice

Ex.  $X =$  number of  $\square$  that show

Probability Distribution

$X$	0	1	2	3
$P(X)$	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

$\rightarrow 1$

$$P(X=0) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{125}{216}$$

$$P(X=1) = 3 \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{75}{216}$$

$$P(X=2) = 3 \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{15}{216}$$

$$P(X=3) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$$

Find the mean (average outcome) of this distribution :

$$\mu = E(X) = \frac{125}{216}(0) + \frac{75}{216}(1) + \frac{15}{216}(2) + \frac{1}{216}(3)$$

↑  
expected value of  $X$  =  $\frac{1}{2}$

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The median is the value that divides the probability into 2 sections of 0.5 each.

$$\text{Median} = 0$$

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The standard Deviation

$$\text{Variance} = \sigma^2 = E(X^2) - [E(X)]^2$$

$$\text{Std. dev.} = \sigma = \sqrt{\sigma^2}$$

$$E(X^2) = \frac{125}{216}(0^2) + \frac{75}{216}(1^2) + \frac{15}{216}(2^2) + \frac{1}{216}(3^2) = \frac{144}{216}$$

$$\sigma^2 = E(X^2) - [E(X)]^2$$

$$= \frac{144}{216} - \left(\frac{1}{2}\right)^2 = \frac{5}{12}$$

Standard deviation:  $\sigma = \sqrt{\frac{5}{12}} \approx \underline{0.645}$

Ex. The probability distribution for  $Y$  is:

$Y$	1	2	3	4	5
$P(Y)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$p$	$3p$

$$4p = \frac{1}{2}$$

$$p = \frac{1}{8}$$


Nathan

a) Find  $p$ .

b) Find  $E(Y)$

c) Find the variance of  $Y$ .

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$$E(Y) = \frac{1}{8}(1) + \frac{1}{8}(2) + \frac{1}{4}(3) + \frac{1}{8}(4) + \frac{3}{8}(5)$$

$$= 3.5 = \text{the mean}$$

$$+ \frac{3}{8}(5^2)$$

$$E(Y^2) = \frac{1}{8}(1^2) + \frac{1}{8}(2^2) + \frac{1}{4}(3^2) + \frac{1}{8}(4^2) + \frac{3}{8}(5^2)$$
$$= 14.25$$

$$\sigma^2 = 14.25 - (3.5)^2 = 2$$

$$\sigma^2 = E(Y^2) - (E(Y))^2$$

Ex. Toss 5 coins. Count the heads.

$X$  = number of heads

$X$	0	1	2	3	4	5
$P(X)$				$\frac{10}{32}$		

$$P(X=3) = \binom{5}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$$

Annotations for the equation above:

- number of trials (points to 5)
- number of successes (points to 3)
- probability of success (points to  $\frac{1}{2}$ )
- number of failures (points to 2)
- probability of failure (points to  $\frac{1}{2}$ )

$$= 10 \left(\frac{1}{2}\right)^5 = \frac{10}{32}$$

HW 15 B # 2, 6