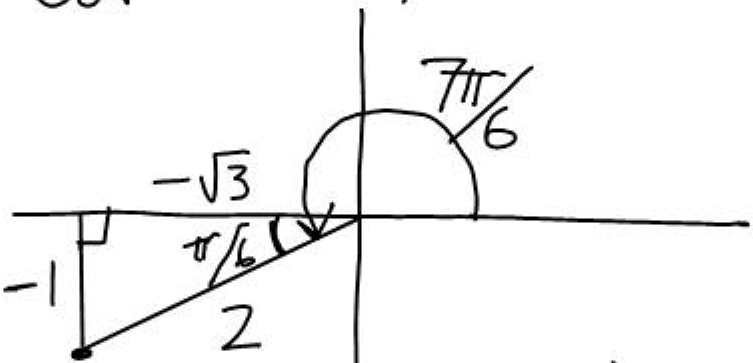
Ex. DeMoivre's Theorem

$$\underbrace{\left(-\sqrt{3} - i\right)^6}_{\text{Cartesian form}} = \left(2 \operatorname{cis} \frac{7\pi}{6}\right)^6$$

modulus–argument form



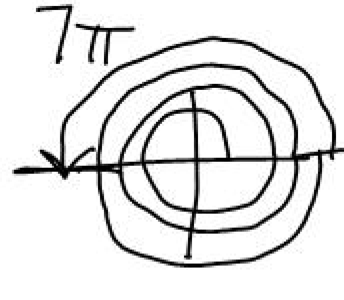
$(-\sqrt{3}, -1) = \left(2, \frac{7\pi}{6}\right)$

Cartesian coordinates

polar coordinates

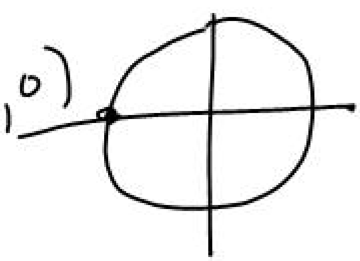
$$= 2^6 \operatorname{cis}\left(\cancel{6} \cdot \frac{7\pi}{\cancel{6}}\right)$$

$$= 64 \operatorname{cis} 7\pi$$



$$= 64 \operatorname{cis} \pi$$

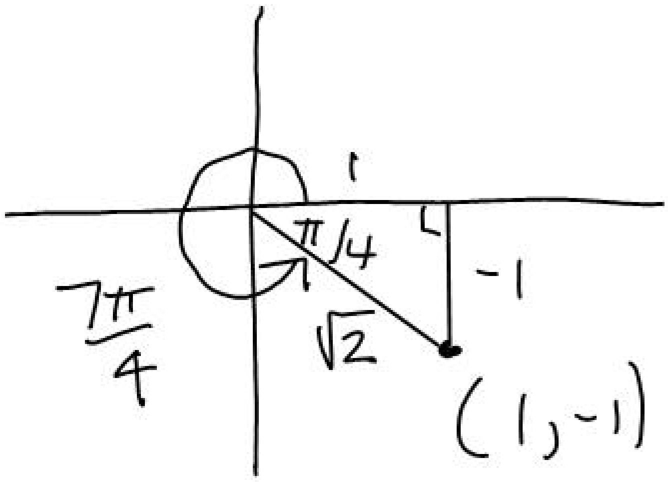
$$= 64\left(\cos\pi + i \cdot \sin\pi\right)$$



$$= 64\left(-1 + i \cdot 0\right)$$

$$= \underline{-64}$$

Ex. $(1-i)^{12} = \left(\sqrt{2}\ \text{cis}\ \frac{7\pi}{4}\right)^{12}$



$(1,-1) = \left(\sqrt{2}, \frac{7\pi}{4}\right)$

$= (\sqrt{2})^{12}\ \text{cis}\left(\cancel{12}^{\,3} \cdot \frac{7\pi}{\cancel{4}}\right)$

$64\ \text{cis}\ 21\pi$

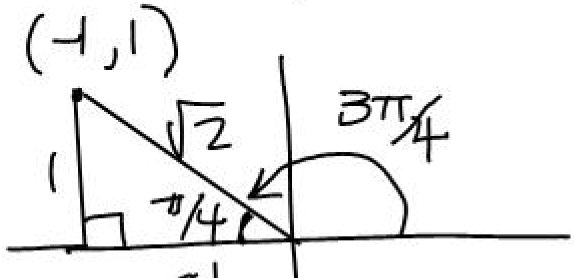
$= 64\ \text{cis}\ \pi$

$= 64\left(\cancel{\cos\pi}^{\,-1} + i \cdot \cancel{\sin\pi}^{\,0}\right)$

$= -64$

Ex. Using DeMoivre's theorem to get <u>roots</u> of complex numbers

Ex. Find the 3 cube roots of $-1+i$



$$-1+i = \sqrt{2}\ \text{cis}\ \frac{3\pi}{4}$$

$$(-1+i)^{\frac{1}{3}} = \left(\sqrt{2}\ \text{cis}\ \frac{3\pi}{4}\right)^{\frac{1}{3}}$$

$$= \sqrt{2}^{1/3}\ \text{cis}\left(\frac{1}{\cancel{3}}\cdot\frac{\cancel{3}\pi}{4}\right)$$

$$\left(2^{\frac{1}{2}}\right)^{\frac{1}{3}} = 2^{\frac{1}{6}}$$

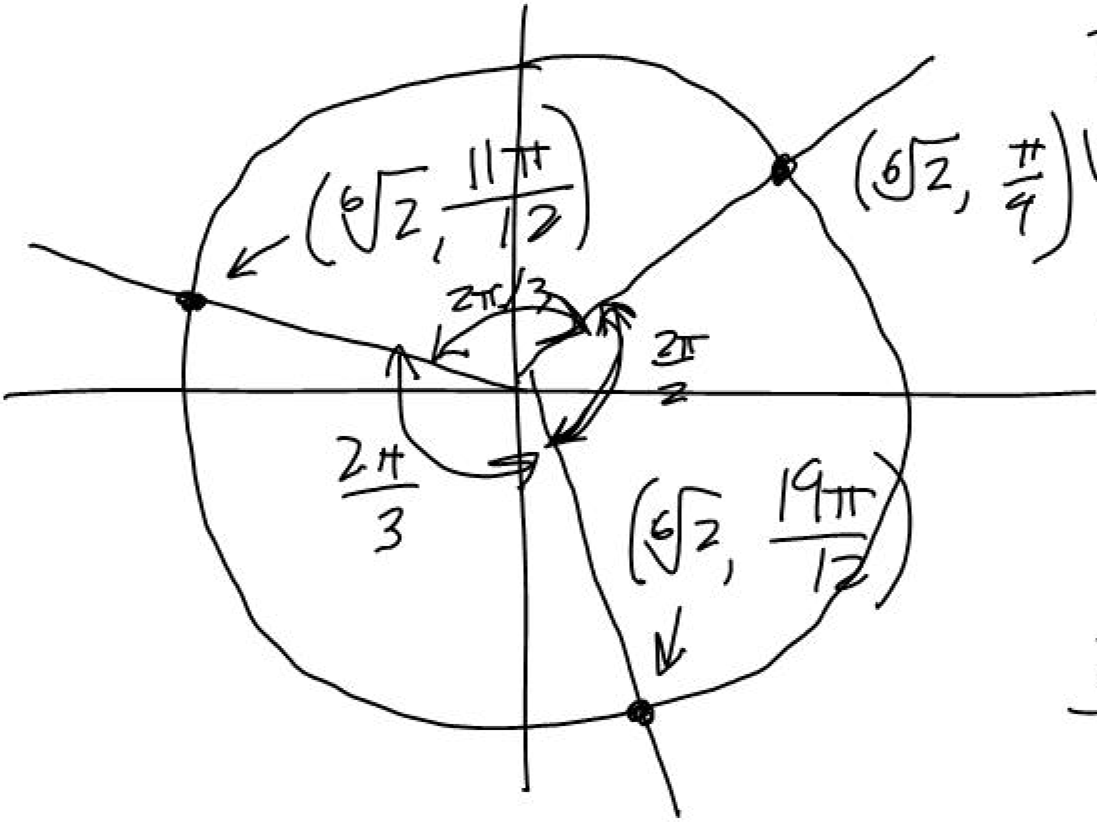
$$= \sqrt[6]{2}\left(\cos\frac{\pi}{4} + i\cdot\sin\frac{\pi}{4}\right) \ast$$

$$\frac{2^{\frac{1}{6}}}{2^{\frac{1}{2}}} = \frac{1}{2^{1/3}}$$

$$= \sqrt[6]{2}\left(\frac{1}{\sqrt{2}} + i\cdot\frac{1}{\sqrt{2}}\right)$$

$$= \boxed{\frac{1}{\sqrt[3]{2}} + \frac{1}{\sqrt[3]{2}}\,i}$$

This is <u>one</u> of the three cube roots



The 3 cube roots are equally spaced on a circle

$$\frac{2\pi}{3} \begin{array}{l} \leftarrow \text{whole circle} \\ \leftarrow \text{3 roots} \end{array}$$

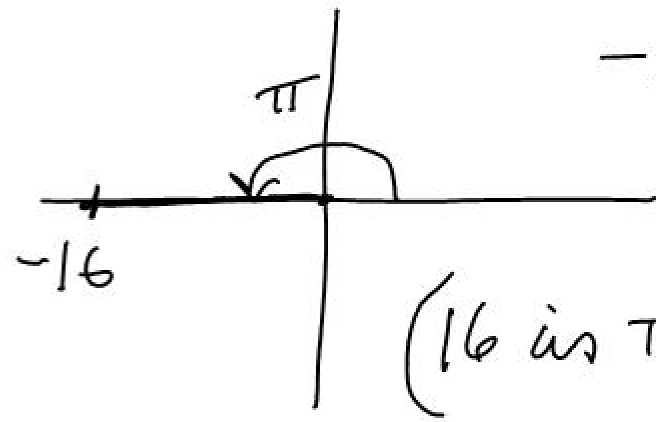
$$\frac{\pi}{4} + \frac{2\pi}{3} = \frac{3\pi}{12} + \frac{8\pi}{12} = \frac{11\pi}{12}$$

$$\frac{11\pi}{12} + \frac{2\pi}{3} = \frac{19\pi}{12}$$

The 3 cube roots of $-1+i$ are

- $\sqrt[6]{2}$ cis $\frac{\pi}{4} = \frac{1}{\sqrt[3]{2}} + \frac{1}{\sqrt[3]{2}}i$
- $\sqrt[6]{2}$ cis $\frac{11\pi}{12}$
- $\sqrt[6]{2}$ cis $\frac{19\pi}{12}$

Ex. Find the 4 fourth roots of -16



$$-16 = 16 \text{ cis } \pi$$

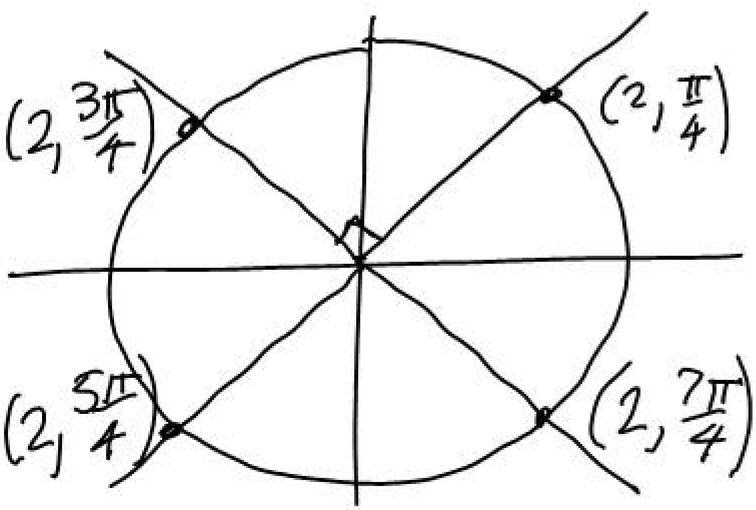
$$\left(16 \text{ cis } \pi\right)^{\frac{1}{4}} = 16^{\frac{1}{4}} \text{ cis}\left(\tfrac{1}{4}\pi\right)$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$= 2\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$$

$$= \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}} + i \cdot \frac{\sqrt{2}}{\cancel{2}}\right)$$

$$= \boxed{\sqrt{2} + i\sqrt{2}}$$



one of the fourth roots

$$\frac{2\pi}{4} = \frac{\pi}{2}$$

The four fourth roots -16 are

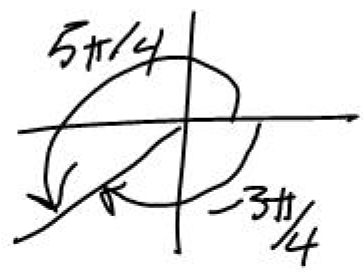
- $2 \text{ cis } \pi/4 = \sqrt{2} + i\sqrt{2}$
- $2 \text{ cis } 3\pi/4 = -\sqrt{2} + i\sqrt{2}$
- $2 \text{ cis } 5\pi/4 = -\sqrt{2} - i\sqrt{2}$
- $2 \text{ cis } 7\pi/4 = \sqrt{2} - i\sqrt{2}$

Powers of $1 + i = \sqrt{2} \operatorname{cis} \frac{\pi}{4}$

$(1 + i)^{-4} = \left(\sqrt{2} \operatorname{cis} \frac{\pi}{4}\right)^{-4} = \sqrt{2}^{-4} \operatorname{cis}(-\pi)$

$= \frac{1}{4} \operatorname{cis} \pi$

$(1+i)^{-3} = \sqrt{2}^{-3} \operatorname{cis} \frac{-3\pi}{4} = \frac{1}{2\sqrt{2}} \operatorname{cis} \frac{5\pi}{4} \approx$



$(1+i)^{-2}$

$(1+i)^{-1}$

$(1+i)^{0} = 1$

$(1+i)^{1} = 1 + i$

$(1+i)^{2}$

$(1+i)^{3}$

$(1+i)^{4}$

$(1+i)^{5}$

$(1+i)^{6}$

$(1+i)^{7}$ $\quad$ $(1+i)^{8}$