The Sine Rule (The Ambiguous Case)

Given: SSA

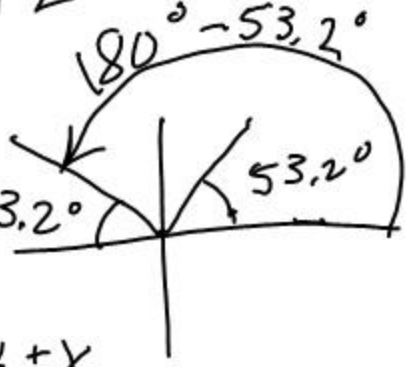
#1 $\alpha = 31°$, $c = 84$, $a = 54$ Find γ.

$$\frac{\sin\gamma}{84} = \frac{\sin 31°}{54}$$

$$\sin\gamma = \frac{84 \cdot \sin 31°}{54} \approx 0.8012$$

quad II

$\gamma = 53.2°$ or $126.8°$



$+ 31$

$157.8 = \alpha + \gamma$

Solution 1

$a = 54$	$\alpha = 31°$	
$b = 104.3$	$\beta = 95.8°$	
$c = 84$	$\gamma = 53.2°$	

$$\frac{\sin 95.8°}{b} = \frac{\sin 31°}{54}$$

Solution 2

$a = 54$	$\alpha = 31°$	
$b = 39.6$	$\beta = 22.2°$	
$c = 84$	$\gamma = 126.8°$	

$$\frac{\sin 22.2°}{b} = \frac{\sin 31°}{54}$$

$$b = \frac{54 \cdot \sin 95.8^\circ}{\sin 31^\circ}$$

$$b = 104.3$$

$$b = \frac{54 \cdot \sin 22.2^\circ}{\sin 31^\circ}$$

$$b = 39.6$$

#2 $\alpha = 31^\circ$, $c = 84$, $a = 85$ Find γ.

$$\frac{\sin \gamma}{84} = \frac{\sin 31^\circ}{85}$$

$$\sin \gamma = \frac{84 \cdot \sin 31^\circ}{85} \approx 0.5090$$

quad I

$\gamma = 30.6^\circ$ or $180 - 30.6^\circ = 149.4^\circ$

quad II

$149.4^\circ + 31$

$\alpha + \gamma = 180.4$

$a = 85$	$\alpha = 31^\circ$
$b = 145.2^\circ$	$\beta = 118.4^\circ$
$c = 84$	$\gamma = 30.6^\circ$

$$\frac{\sin 118.4^\circ}{b} = \frac{\sin 31^\circ}{85}$$

$$b = 145.2^\circ$$

#3 $\alpha = 31^\circ$, $c = 84$, $a = 40$ Find γ

$$\frac{\sin\gamma}{84} = \frac{\sin 31^\circ}{40}$$

$$\sin\gamma = \frac{84 \cdot \sin 31^\circ}{40} = 1.08$$

NO SUCH TRIANGLE

#9 $\alpha = 40^\circ$, $a = 15$, $b = 14$ Find β

$$\frac{\sin\beta}{14} = \frac{\sin 40^\circ}{15}$$

$$\sin\beta = \frac{14 \sin 40^\circ}{15} \approx 0.5999$$

$\boxed{\beta = 36.9^\circ}$ or ~~143.1°~~

~~40~~

~~183.1~~

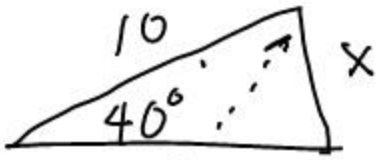
#13 $\beta = 80°$, $b = 10$, $c = 9$ Find γ

$$\frac{\sin\gamma}{9} = \frac{\sin 80°}{10}$$

$\sin\gamma \approx 0.8863$

$\boxed{\gamma = 62.4°}$ or $180 - 62.4$

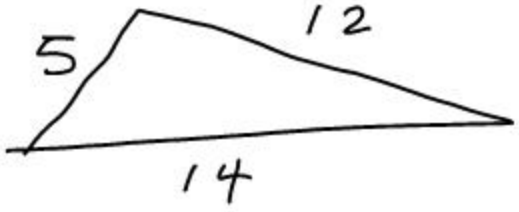
SAS



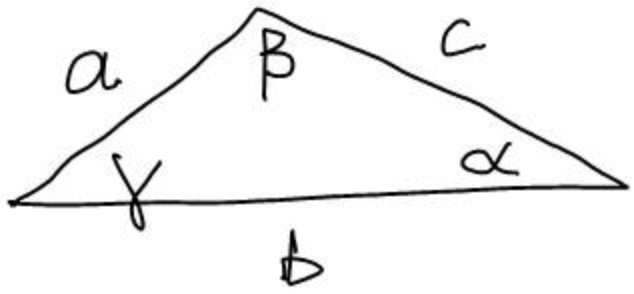
$$\frac{\sin 40°}{x} = \frac{\quad}{\cancel{10}\ 12}$$

We can't use the sine rule

SSS



The Cosine Rule (The Law of Cosines)



$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

(opposites)

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

SAS

Solve for $\cos\gamma$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 - a^2 - b^2 = -2ab\cos\gamma$$

$$\frac{c^2 - a^2 - b^2}{-2ab} = \cos\gamma$$

SSS

$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

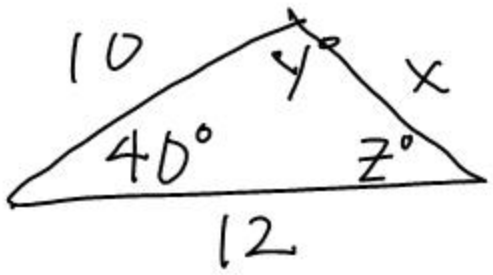
opposite

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

Ex

SAS



$$x^2 = 10^2 + 12^2 - 2(10)(12)\cos 40^\circ$$

$$x^2 = 60.149$$

$$x = 7.8 \quad (7.7556)$$

Find y:

$$\frac{\sin y^\circ}{12} = \frac{\sin 40^\circ}{7.7556}$$

$$\sin y = 0.9946$$

$$y = 84.0^\circ$$

$$z = 56.0^\circ$$