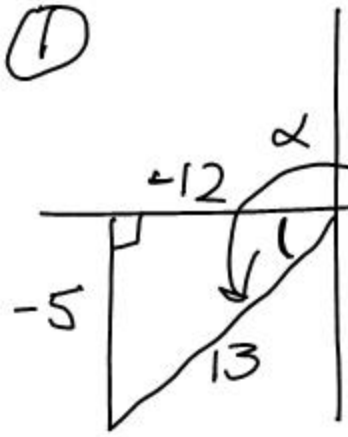
(1)



[a] $\cos\alpha = \frac{-12}{13}$

[b] $\sin 2\alpha = 2\sin\alpha\cos\alpha$

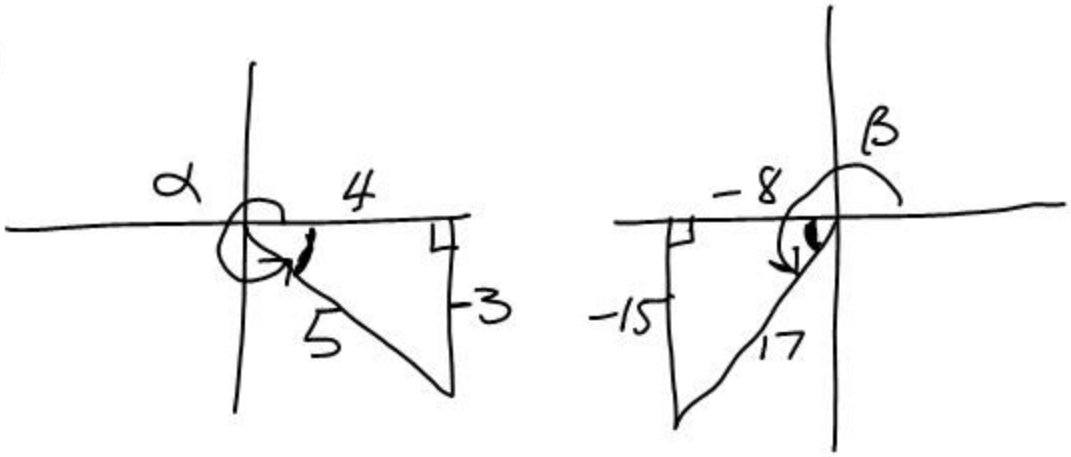
$= 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)$

$= \frac{120}{169}$

[c] $\sin\frac{\alpha}{2} = \sqrt{\frac{1}{2} - \frac{1}{2}\cos\alpha} = \sqrt{\frac{1}{2} - \frac{1}{2}\left(\frac{-12}{13}\right)}$

$= \sqrt{\frac{1}{2} + \frac{12}{26}} = \sqrt{\frac{13}{26} + \frac{12}{26}} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}}$

(2)



[a] $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$= \left(\frac{-3}{5}\right)\left(\frac{-8}{17}\right) + \left(\frac{-15}{17}\right)\left(\frac{4}{5}\right)$

$= \frac{24}{85} + \frac{-60}{85} = \frac{-36}{85}$

(2)[b] $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(\frac{4}{5}\right)\left(\frac{-8}{17}\right) + \left(\frac{-3}{5}\right)\left(\frac{-15}{17}\right)$

$= \frac{-32}{85} + \frac{45}{85} = \frac{13}{85}$

(3) $\sin\frac{11\pi}{8} = \sin\frac{\left(\frac{11\pi}{4}\right)}{2} = -\sqrt{\frac{1}{2} - \frac{1}{2}\cos\frac{11\pi}{4}}$

quad III, sine is negative

coterminal with $\frac{3\pi}{4}$

$= -\sqrt{\frac{1}{2} - \frac{1}{2}\left(\frac{-\sqrt{2}}{2}\right)}$

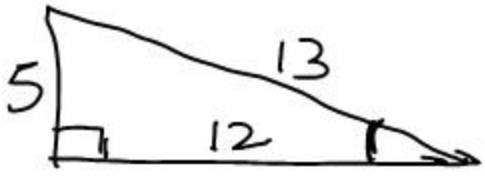
$= -\sqrt{\frac{1}{2} + \frac{\sqrt{2}}{4}} = -\sqrt{\frac{2+\sqrt{2}}{4}} = \frac{-\sqrt{2+\sqrt{2}}}{2}$

(4)[a] $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

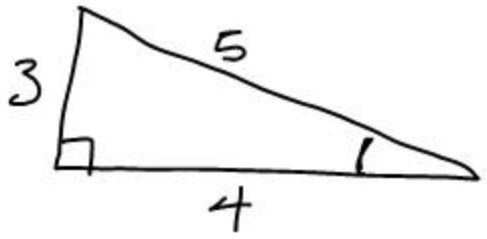
[b] $\cos^{-1}\left(\cos\frac{4\pi}{3}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

[c] $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

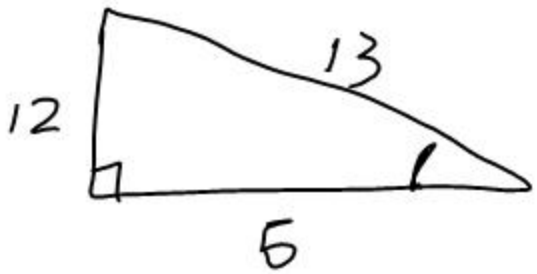
[e] $\csc\left(\cos^{-1}\frac{12}{13}\right) = \frac{13}{5}$



(5) [a] $\cos\left(\sin^{-1}\frac{3}{5}\right) = \frac{4}{5}$



[b] $\tan\left(\cos^{-1}\frac{5}{13}\right) = \frac{12}{5}$



(6) [a] $\cos\alpha = \frac{1}{\sqrt{2}} \Rightarrow$ quad I, quad IV

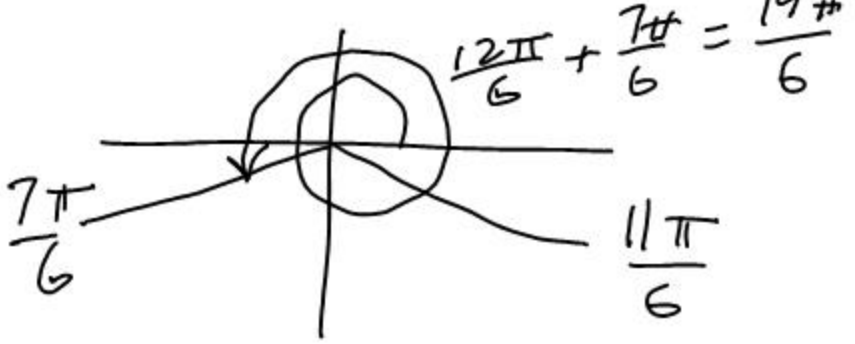
$$\alpha = \frac{\pi}{4} \text{ or } \frac{7\pi}{4}$$

[b] $\sin 2\beta = -\frac{1}{2}$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$$2\beta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$$

$$\beta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$



[c] $4\cos^2\theta = 3$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow$ Q I, Q II, Q III, Q IV

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

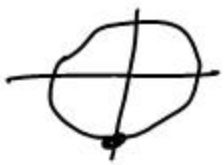
[d] $2\sin^2\theta + \sin\theta - 1 = 0$

$2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$y = \frac{1}{2}$ or $y = -1$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$ →



$\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ or $\frac{3\pi}{2}$

6 [e] $2\sin^2\theta - 5\cos\theta - 4 = 0$

$2(1-\cos^2\theta) - 5\cos\theta - 4 = 0$

$2 - 2\cos^2\theta - 5\cos\theta - 4 = 0$

$-2\cos^2\theta - 5\cos\theta - 2 = 0$

$2\cos^2\theta + 5\cos\theta + 2 = 0$

$2y^2 + 5y + 2 = 0$

$(2y + 1)(y + 2) = 0$

$y = -1/2$ or $y = -2$

$\cos\theta = -1/2$ or ~~$\cos\theta = -2$~~ no solutions

$\theta = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

QII QIII

① [a] $\dfrac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$

$\dfrac{2\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} \rightarrow \dfrac{\frac{2\sin\theta}{\cos\theta}}{\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}}$

$\rightarrow \dfrac{\frac{2\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}} \rightarrow \dfrac{2\sin\theta}{\cancel{\cos\theta}} \cdot \dfrac{\cos^{\cancel{2}}\theta}{1}$

$\rightarrow 2\sin\theta\cos\theta \rightarrow \sin 2\theta$

[b] $(\sin\theta+\cos\theta)^2 = 1+\sin 2\theta$

$(\sin\theta+\cos\theta)(\sin\theta+\cos\theta)$

$\rightarrow \underline{\sin^2\theta} + \sin\theta\cos\theta + \cos\theta\sin\theta \underline{+\cos^2\theta}$

$\rightarrow 1 + 2\sin\theta\cos\theta$

$\rightarrow 1 + \sin 2\theta$

[c] $\frac{\sec^2\theta}{\tan\theta} = \sec\theta\csc\theta$

$\frac{1}{\cos^2\theta}$ / $\frac{\sin\theta}{\cos\theta}$ $\rightarrow \frac{1}{\cos^2\theta}\cdot\frac{\cos\theta}{\sin\theta} \rightarrow \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$

$\rightarrow \sec\theta\cdot\csc\theta$

[d] $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \rightarrow \frac{\sin\theta\sin\theta}{\cos\theta\sin\theta} + \frac{\cos\theta\cos\theta}{\sin\theta\cos\theta}$

$\rightarrow \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} \rightarrow \frac{1}{\sin\theta\cos\theta}$

$\rightarrow \frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta} \rightarrow \csc\theta\cdot\sec\theta$

Identities to Know & Love

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 1 - 2\sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$\sin\left(\frac{1}{2}\theta\right) = \pm\sqrt{\frac{1}{2} - \frac{1}{2}\cos\theta}$$

$$\cos\left(\frac{1}{2}\theta\right) = \pm\sqrt{\frac{1}{2} + \frac{1}{2}\cos\theta}$$