

# Two kinds of trig problems from Geometry

① Given 2 sides of a right triangle, find the acute angles.

Ex.

$$\cos x = \frac{7}{10}$$
$$x = \cos^{-1} \left( \frac{7}{10} \right)$$
$$= 45.6^\circ$$

$$\text{sine} = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

SOH

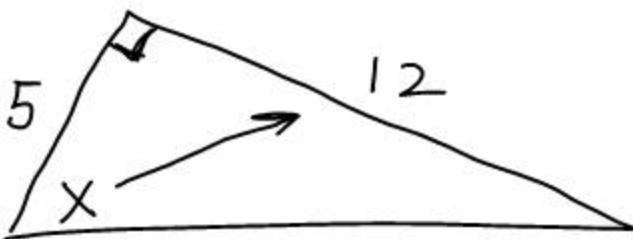
$$\text{cosine} = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

CAH

$$\text{tangent} = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

TOA

Ex.

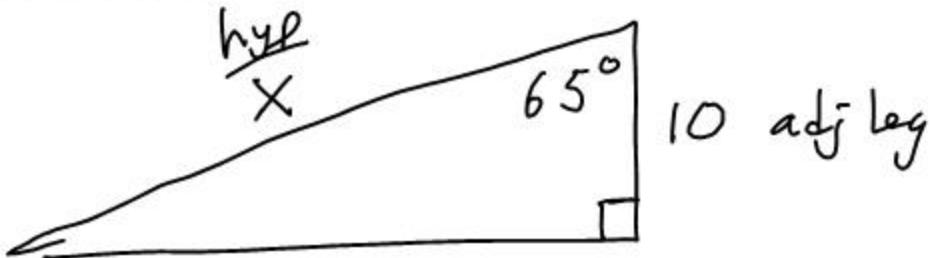


$$\tan x = \frac{12}{5}$$

$$x = 67.4^\circ$$

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- ② Given 1 side and an acute angle  
in a right triangle, find the other  
sides.
- 

Ex.

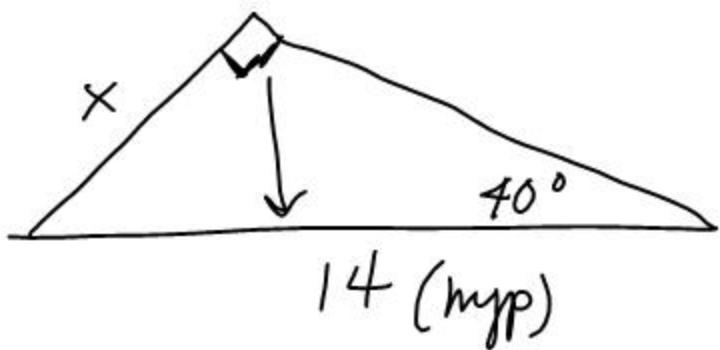


$$\cos 65^\circ = \frac{10}{x}$$

$$x \cdot \cos 65^\circ = 10$$

$$x = \frac{10}{\cos 65^\circ} = 23.7$$

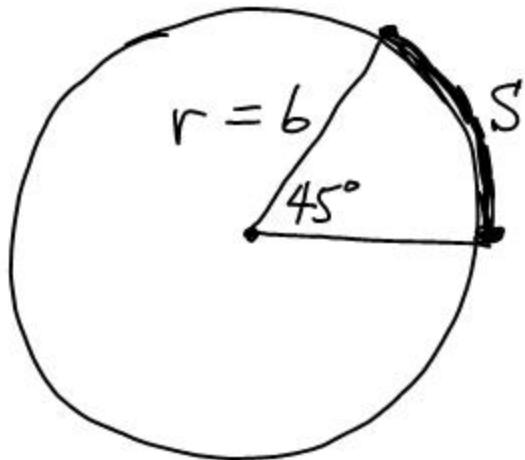
Ex.



$$\sin 40^\circ = \frac{X}{14}$$

$$X = 14 \cdot \sin 40^\circ = 9.0$$

## Radian Measure for Angles



Convert the  $45^\circ$  angle measure to radian measure.

$S = \text{arc length}$

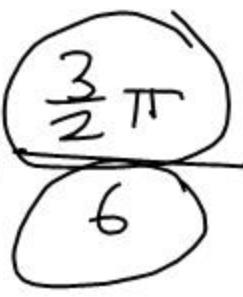
$$= \underbrace{\frac{45}{360} \cdot 2\pi \cdot 6}_C$$

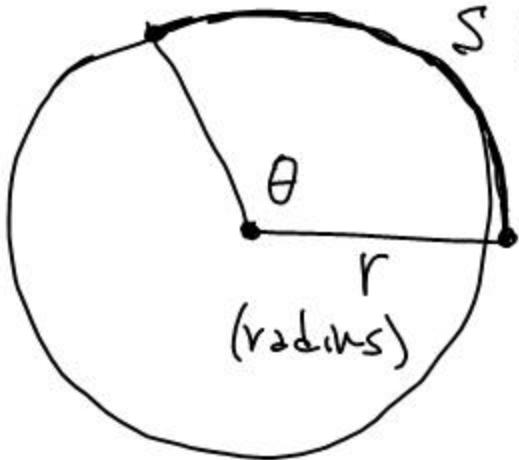
fraction  
of the circumference

$$S = \frac{3}{2}\pi$$

The radian measure of the  $45^\circ$  angle is

$$=\frac{\pi}{4}$$



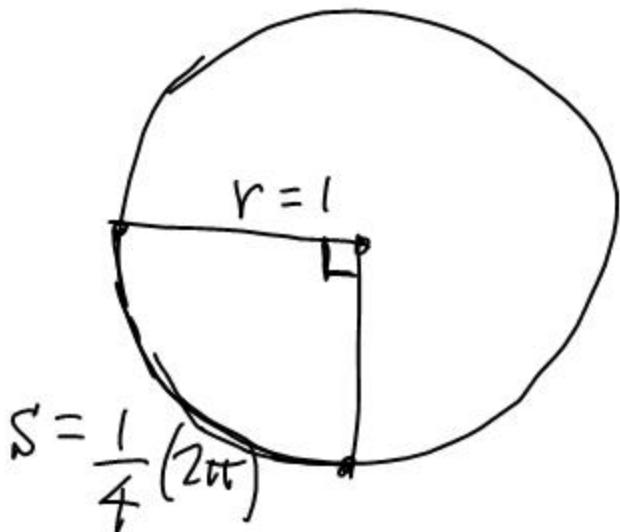


$S$  (arc length)

$\theta$  theta  
 $\alpha$  alpha  
 $\beta$  beta  
 $\gamma$  gamma

$$\theta = \frac{S}{r}$$

Ex. Convert  $90^\circ$  to radians.



$$S = \frac{1}{4}(2\pi)$$

$$= \frac{\pi}{2}$$

$$90^\circ = \frac{\pi}{2}$$

$$180^\circ = \pi$$

Conversion factors:  $\frac{180^\circ}{\pi}$  or  $\frac{\pi}{180^\circ}$

Convert  $15^\circ$  to radians.

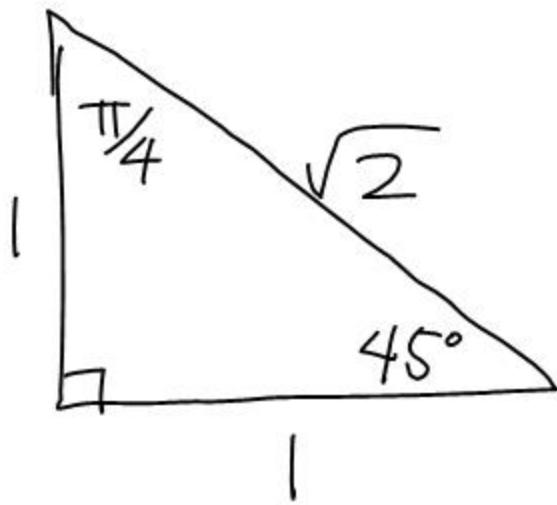
$$15^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{12}$$

Convert  $\frac{5\pi}{6}$  to degrees.

$$\frac{5\pi}{4} \cdot \frac{180^\circ}{\pi} = 150^\circ$$

degrees	radians	degrees	radians
$0^\circ$	0	$180^\circ$	$\pi$
$30^\circ$	$\pi/6$	$210^\circ$	$7\pi/6$
$45^\circ$	$\pi/4$	$225^\circ$	$5\pi/4$
$60^\circ$	$\pi/3$	$240^\circ$	$4\pi/3$
$90^\circ$	$\pi/2$	$270^\circ$	$3\pi/2$
$120^\circ$	$2\pi/3$	$300^\circ$	$5\pi/3$
$135^\circ$	$3\pi/4$	$315^\circ$	$7\pi/4$
$150^\circ$	$5\pi/6$	$330^\circ$	$11\pi/6$

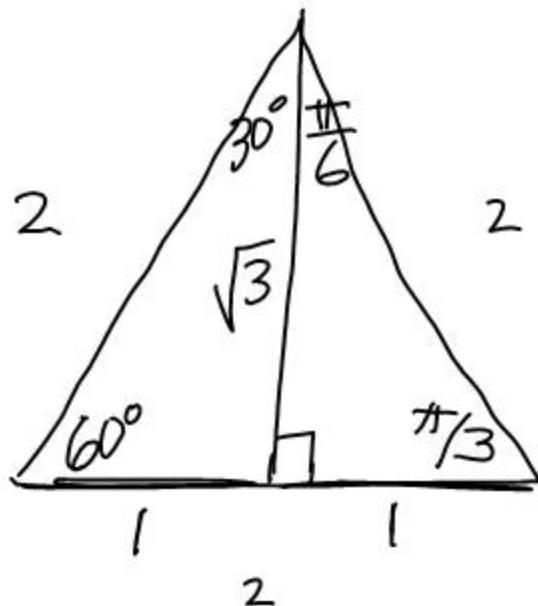
## 2 special right triangles



$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\tan \frac{\pi}{4} = 1$$



$$2^2 = 1^2 + h^2 \Rightarrow h = \sqrt{3}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$

# The "Missing" Trig Ratios

cotangent =  $\frac{\text{adjacent leg}}{\text{opposite leg}}$  reciprocal of tangent  
[cot]

secant =  $\frac{\text{hypotenuse}}{\text{adjacent leg}}$  reciprocal of cosine  
[sec]

cosecant =  $\frac{\text{hypotenuse}}{\text{opposite leg}}$  reciprocal of sine  
[csc]

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$$\cdot \cot \frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

$$\cdot \sec \frac{\pi}{4} = \sqrt{2}$$

$$\cdot \csc \frac{\pi}{6} = 2$$