$$\log_3 \sqrt{243} = \log_3 \left(3^{5/2}\right) = \frac{5}{2}$$

#38. $e^{2\ln 7}$

$= (e^{\ln 7})^2$

$= 7^2$

$= 49$

OR

$a^m \cdot a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

$e^{2\ln 7} = e^{\ln 7^2} = e^{\ln 49}$

$= 49$

#31. $10^{\log 45} = 10^{\log_{10} 45} = 45$

#67 $\log_8(x+5) - \log_8(x-2) = 1$

log form: $\log_8 \left(\frac{x+5}{x-2}\right) = 1$

expo form: $8^1 = \frac{x+5}{x-2}$

$8x - 16 = x + 5$

$7x = 21$

$\boxed{x = 3}$

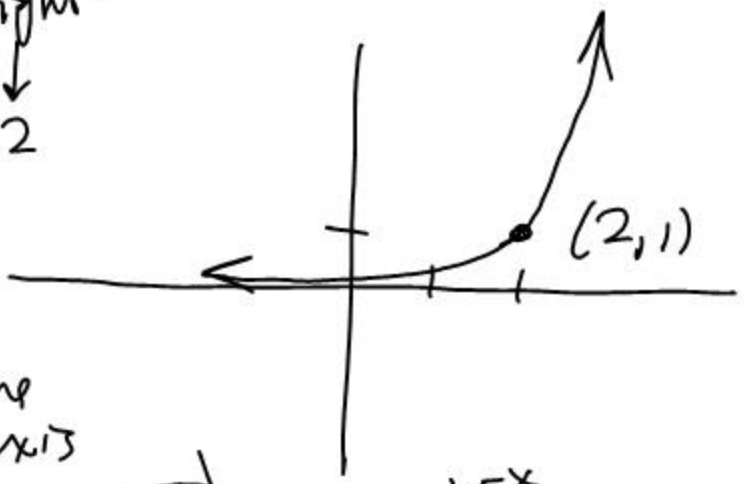
#46. $\log\left(x \cdot \sqrt{x^2+1}\right)$

$\log x + \log \sqrt{x^2+1}$

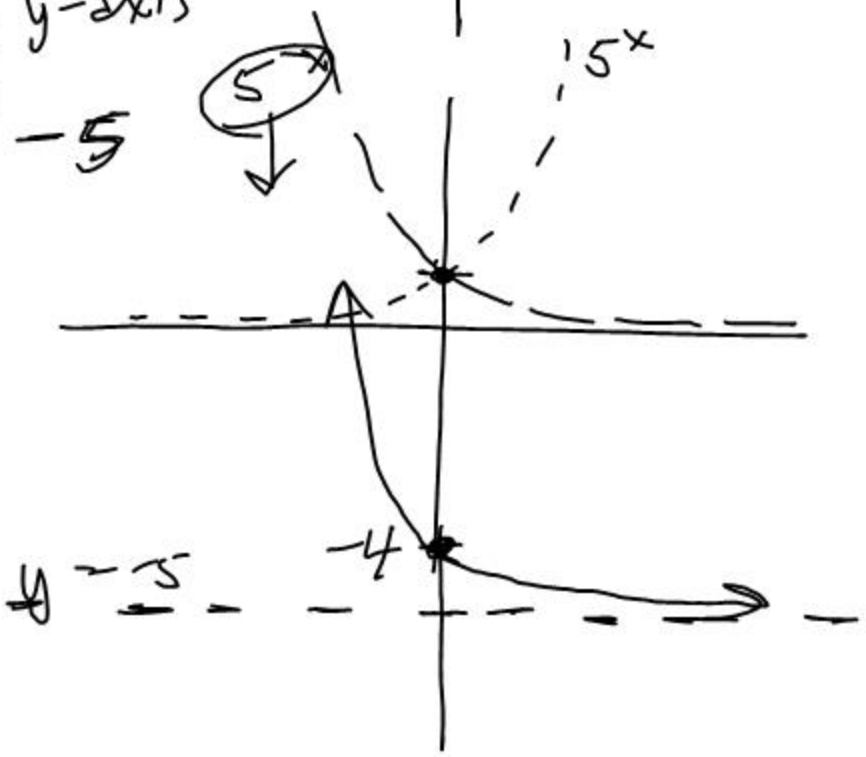
$\log x + \frac{1}{2}\log(x^2+1)$

Graphs

#6. $f(x) = 3^{x-2}$ (right)

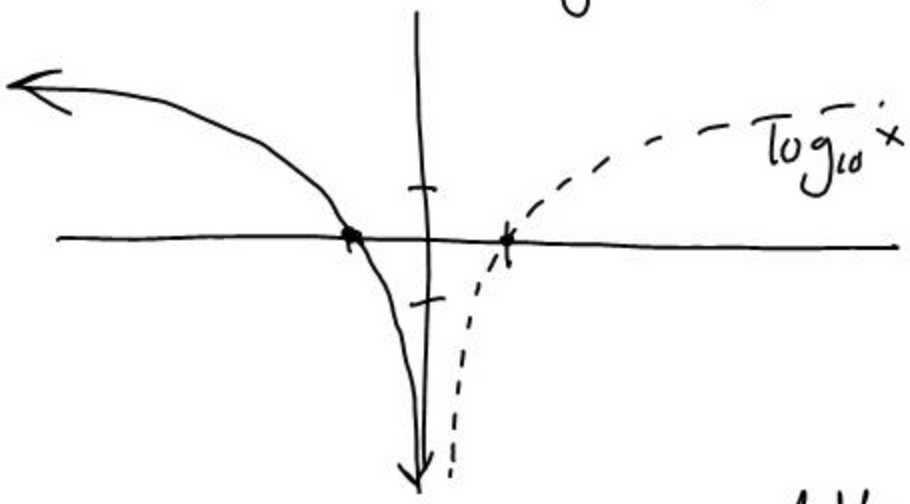


#8. $f(x) = 5^{-x} - 5$ (flip over the y-axis)



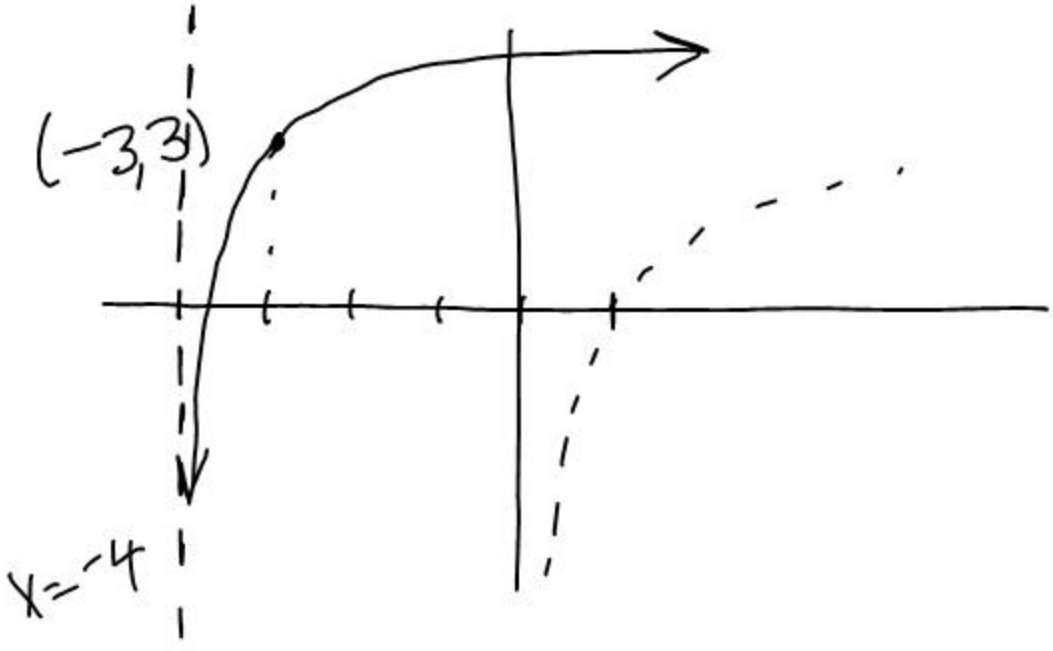
* 10. $g(x) = \log(-x)$

↑ flip over y-axis



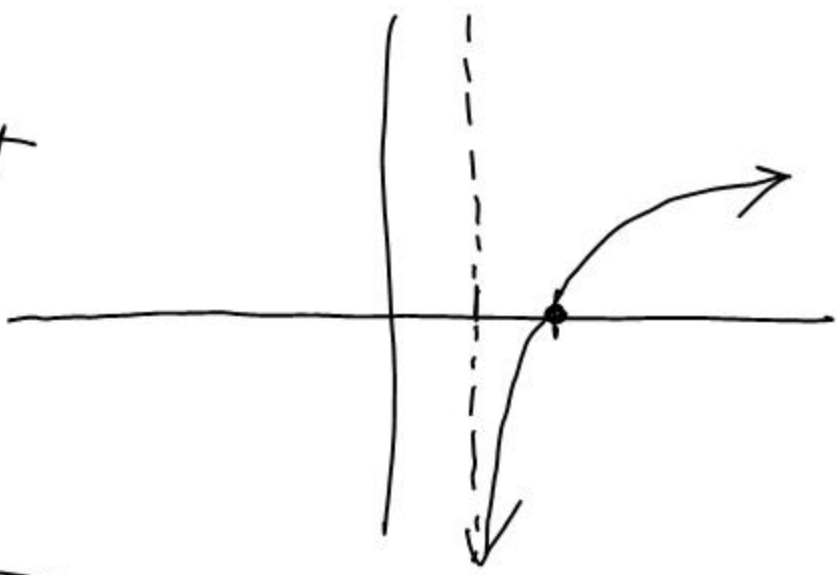
*12. $f(x) = 3 + \log_5(x+4)$

↑ up ← left

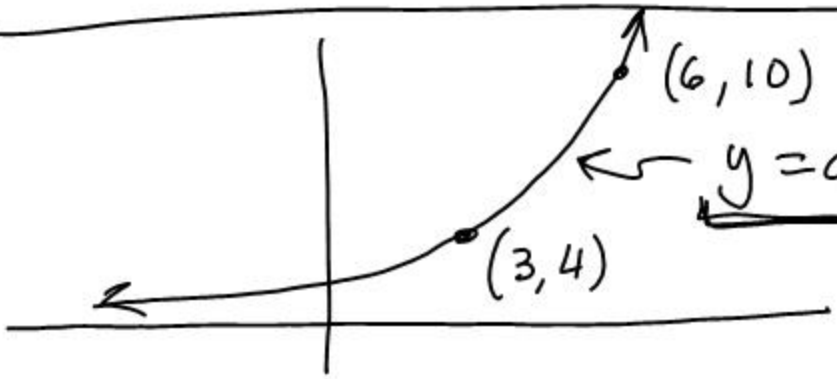


#9, $y = \log_3(x-1)$

↑ right



Test = book review + word problems



$y = a \cdot b^x$ or $y = a \cdot e^{kx}$

natural exponential

$y = a \cdot b^x$

$$\begin{cases} 4 = a \cdot b^3 \longrightarrow a = 4b^{-3} \\ 10 = a \cdot b^b \end{cases}$$

$10 = (4b^{-3}) \cdot b^b = 4b^3$

$b = \left(\frac{5}{2}\right)^{1/3}$

$a = 4\left[\left(\frac{5}{2}\right)^{1/3}\right]^{-3}$

$= 4\left(\frac{5}{2}\right)^{-1}$

$= 4\left(\frac{2}{5}\right) = \frac{8}{5}$

$$y = a \cdot b^x$$

$$\begin{cases} 4 = a \cdot b^3 \\ 10 = a \cdot b^6 \end{cases} \longrightarrow a = 4b^{-3}$$

$$10 = (4b^{-3}) \cdot b^6 = 4b^3$$

$$b = \left(\frac{5}{2}\right)^{1/3}$$

$$a = 4\left[\left(\frac{5}{2}\right)^{1/3}\right]^{-3}$$

$$= 4\left(\frac{5}{2}\right)^{-1}$$

$$= 4\left(\frac{2}{5}\right) = \frac{8}{5}$$

$$y = \frac{8}{5}\left(\frac{5}{2}\right)^{\frac{x}{3}}$$

← convert to natural exponential

$$e^{kx} = \left(\left(\frac{5}{2}\right)^{1/3}\right)^x$$

exponential equation

$$e^k = \left(\frac{5}{2}\right)^{1/3}$$ ← solve for k

$$k \cdot \ln e^k = \ln\left(\frac{5}{2}\right)^{1/3}$$

$$k = \frac{1}{3}\ln\left(\frac{5}{2}\right) \approx 0.3054$$

$$y = \frac{8}{5}e^{0.3054x}$$

Convert to natural exponential form:

$$y = 5 \cdot \left(\frac{3}{8}\right)^x \qquad y = a \cdot e^{kx}$$

$$e^k = \frac{3}{8} \qquad y = 5\, e^{-0.9808x}$$

$$k = \ln\left(\frac{3}{8}\right)$$

$$= -0.9808$$

negative k-value $\Rightarrow$ decay

Ex. A population is 10000 at time $t = 0$. If it grows at 10% per year, write an equation for pop. P in the form

$$P = P_0 e^{kt}.$$

$$P = a \cdot b^t$$

$$P = 10000\,(1 + 0.10)^t$$

$$P = 10000\,(1.1)^t$$

$$e^k = 1.1$$

$$k = \ln 1.1$$

$$= 0.09531$$

$$P = 10000\, e^{0.09531 \cdot t}$$

Ex. Find the rate of growth: (t in years)

$$P = 1000\, e^{0.05t} = a \cdot b^t$$

$$b = e^{0.05} = 1.0513 = 1 + r$$

$$r = 0.0513 = 5.13\%$$

Ex. A pop. begins at 10000 with a rate of growth of 50% per days. How long will it take to reach 125000?

$$P = 10000 \cdot (1.5)^t$$

$$125000 = 10000\,(1.5)^t$$

$$12.5 = 1.5^t$$

$$\ln 12.5 = t \cdot \ln 1.5$$

$$\frac{\ln 12.5}{\ln 1.5} = t = 6.23 \text{ days}$$

Ex: The half-life of aspirin in your bloodstream is 12 hours. If the initial dose is 1000 mg, how much is left after 6 hours?

$$m = a \cdot b^t$$

$$m = 1000\, b^t$$

$$500 = 1000\, b^{12}$$

$$\frac{1}{2} = b^{12}$$

$$b = \left(\frac{1}{2}\right)^{1/12} \approx 0.9439$$

$$m = 1000(0.9439)^t$$

$$m(6) = 1000(0.9436)^6 \approx 707 \text{ g}$$