Ex. $\underbrace{\log_2 x + \log_2(x+4)} = 5$

<u>log form</u> $\log_2(x^2+4x) = 5$

<u>exp form</u>: $2^5 = x^2+4x$

$0 = x^2+4x-32$

$= (x+8)(x-4)$

$x=-8$ or $\boxed{x=4}$

Ex. $6^x = 8^{x+3}$

$\ln 6^x = \ln 8^{x+3}$ $\qquad$ $a\log b = \log b^a$

$x \cdot \ln 6 = (x+3)\cdot \ln 8$

$x\ln 6 = x\ln 8 + 3\ln 8$

$\cancel{x}\ln 6 - \cancel{x}\ln 8 = 3\ln 8$

$x(\ln 6 - \ln 8) = 3\ln 8$

$$\boxed{x = \frac{3\ln 8}{\ln 6 - \ln 8}}$$

Expand: $\ln\left(\dfrac{x^2y^3}{z^4\sqrt{w}}\right)$

$$= \ln\left(x^2y^3\right) - \ln\left(z^4\sqrt{w}\right)$$

$$= 2\ln x + 3\ln y - \left(4\ln z + \tfrac{1}{2}\ln w\right)$$

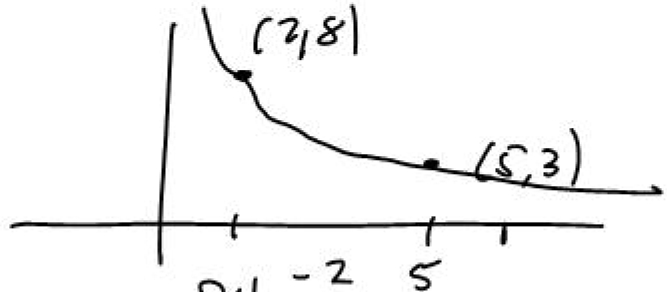
Condense: $\underbrace{\ln 2 - \ln p} + \ln q - 4\ln r$

$$\underbrace{\ln\left(\frac{2}{p}\right) + \ln q} - 4\ln r$$

$$\ln\left(\frac{2q}{p}\right) - \ln r^4$$

$$\ln\left(\frac{2q}{pr^4}\right)$$

$y = a \cdot b^x$



$$\begin{cases} 8 = a \cdot b^2 \longrightarrow a = 8b^{-2} \\ 3 = a \cdot b^5 \end{cases}$$

$3 = 8b^{-2} \cdot b^5 = 8b^3$

$b^3 = \frac{3}{8}$

$\boxed{b = \frac{3^{1/3}}{2}}$

$a = 8\left(\frac{3^{1/3}}{2}\right)^{-2}$

$a = 32(3)^{-2/3}$

$\boxed{a = \frac{32}{3^{2/3}}}$

$\boxed{y = \frac{32}{3^{2/3}}\left(\frac{3^{1/3}}{2}\right)^x}$

$\boxed{y = 15.384\,(\underline{0.7211})^x}$

base = 1 + growth rate

$1 + r = 0.7211$

$r = -0.2789$ or -27.89%

growth rate

ⓒ $\underline{f(6) = 2.16}$

Population

The initial pop. of a town is 5000.
The town is growing at 1.5% each year.

(a) Find the pop. in 8 years.

(b) How long until the pop. reaches 6000?

$$P = a \cdot b^t$$

a: pop. at time 0

$b = 1 + r$

$1 + 1.5\%$

$$\boxed{P = 5000(1.015)^t}$$

a $P(8) = 5000(1.015)^8 = 5632$

b $6000 = 5000(1.015)^t$ ← solve for t in the exp. eq.

$$\frac{6}{5} = (1.015)^t$$

$$\ln\left(\frac{6}{5}\right) = t \cdot \ln 1.015$$

$$t = \frac{\ln(6/5)}{\ln 1.015} = \underline{12.2 \text{ yrs.}}$$

Ex. Some element has a half-life of 152 days. Start with 100g of the material.

a) How much is left after 100 days?

b) How long will it take to decay to 10 g?

$$m = a \cdot b^t$$

a: 100g

$b = 1+r$

$$m = 100 \cdot b^t$$

$$50 = \cancel{100} \cdot b^{152}$$

$$\frac{1}{2} = b^{152}$$

$$b = \left(\frac{1}{2}\right)^{\frac{1}{152}} \approx 0.99545$$

$$\boxed{m = 100(0.99545)^t}$$

$1+r = 0.99545$

$\underline{r = -0.00455}$

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a) $m(100) = 100(0.99545)^{100} = 63.4\text{g}$

b) $10 = 100(0.99545)^t$

$\ln 0.1 = t \ln 0.99545 \rightarrow t = \dfrac{\ln 0.1}{\ln .99545}$

≈ 505 days