#69 $x^5 + 4x^3 - x^2 + 6x$ — 2 or 0 pos.

$(-x)^5 + 4(-x)^3 - (-x)^2 + 6(-x)$

$-x^5 - 4x^3 - x^2 - 6x$ — no neg roots

~~3~~ reals + ~~2~~ non-reals

or

1 ~~0~~ reals + ~~4~~ non-reals

#36 $\frac{1}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i}{1 - \cancel{i^2}(-1)}$

$= \frac{1}{2} - \frac{1}{2}i$

#34 $i^{1002} = i^{1000} \cdot i^2 = \cancel{(i^4)^{250}} \cdot i^2 = -1$

1

$(i^3)^{334} = (-i)^{334} = i^{334} = \cancel{i^{332}} \cdot i^2$

1

#43. $\dfrac{4+6i}{3i} \cdot \dfrac{-3i}{-3i} = \dfrac{+18i - 12i}{-9i^2 \; 9}$

$= 2 - \frac{4}{3}i$

$0+3i$
conjugate: $0-3i$

Ex. $i^{3047} = i^{3044} \cdot i^3 = i \cdot i^2 = -i$

$3044 = 4 \cdot 761$

$(i^4)^{761}$

Ex. $i^{914} = i^{912} \cdot i^2 = -1$

Ex. $\sqrt{8-6i}$

$\sqrt{8-6i} = a+bi$

$8-6i = (a+bi)^2$

$8-6i = a^2 + 2abi + \cancel{\pm b^2} - b^2$

Ex.

$8-6i = (a^2-b^2) + (2ab)i$

$$\begin{cases} a^2-b^2 = 8 \\ 2ab = -6 \longrightarrow b = \frac{-6}{2a} \rightarrow \boxed{b = \frac{-3}{a}} \end{cases}$$

$$a^2 - \left(\frac{-3}{a}\right)^2 = 8$$

$$a^2\left[a^2 - \frac{9}{a^2}\right] = 8 \;]\, a^2$$

$$a^4 - 9 = 8a^2$$

$$a^4 - 8a^2 - 9 = 0$$

$$(a^2+1)(a^2-9) = 0$$

a^2+1 ✗

$a = 3$ or $a = -3$

$b = -1$ or $b = 1$

$\sqrt{8-6i}$

$= 3-i$

or $-3+i$

check

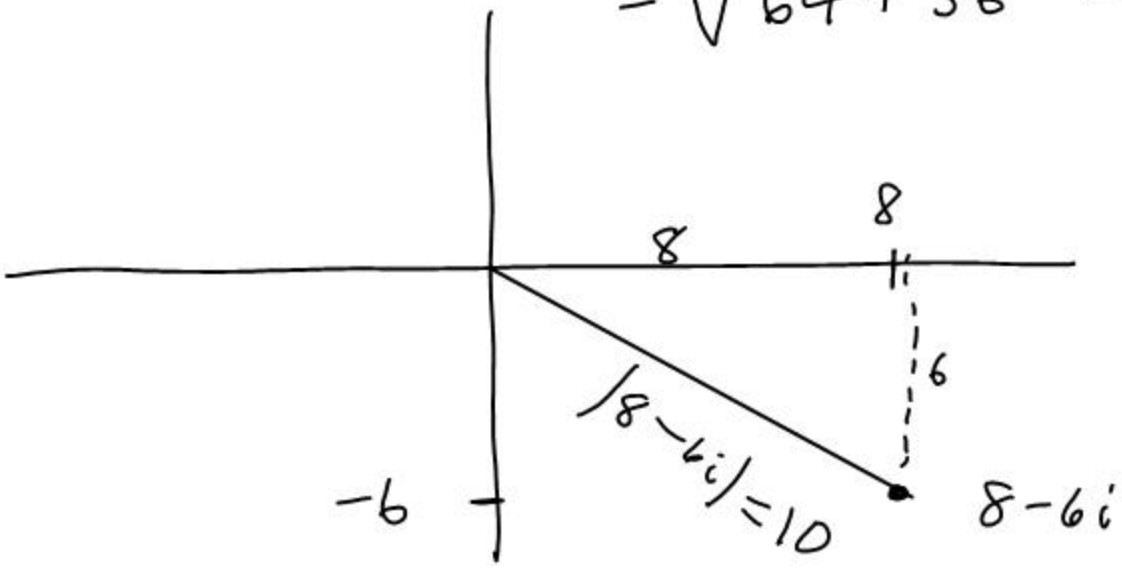
$$(3-i)^2 = (3-i)(3-i) = 9 - 6i + \cancel{i^2}^{-1}$$

$$= 8 - 6i$$

Ex. $|8-6i| = \sqrt{8^2 + (-6)^2}$

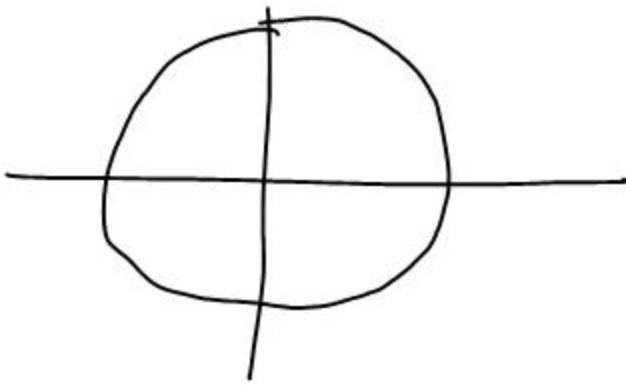
$$= \sqrt{64+36} = \sqrt{100} = 10$$

The modulus of $8-6i$



Ex. $|8+6i| = 10$

$|-8-6i| = 10$



§3.6 #50 $P(x) = x^3 - x - 6$

Find all zeros.

(2)

	1	0	−1	−6
		2	4	6
	1	2	3	0

$(x-2)(x^2+2x+3) = 0$

$x = 2$

$4 - 12 = -8$

$$x = \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm 2\sqrt{2}i}{2}$$

$x = -1 \pm \sqrt{2}\,i$

$-1 \pm i\sqrt{2}$

$\sqrt{-8}$

$\sqrt{(-1)(4)(2)}$

$2\ \sqrt{2}\ i$

#53. $P(x) = x^4 + x^3 + 7x^2 + 9x - 18$

1	1	7	9	−18
	1	2	9	18
	1	2	9	18 | 0

No more positive zeros

−2	1	2	9	18
		−2	0	−18
	1	0	9	0

$(x-1)(x+2)(x^2+9) = 0$

$x = 1$ $\quad x = -2$

$x^2 = -9$

$x = \pm\sqrt{-9}$

$x = \pm 3i$

Ex. Find a 4th degree polynomial with these zeros: $x = 4 - 2i$ and $x = 1 + \sqrt{2}$

$x = 4 - 2i$

$x - 4 = -2i$ ← get i term by itself

$(x-4)^2 = (-2i)^2$

$4(-1)$

$x^2 - 8x + 16 = -4$

$x^2 - 8x + 20 = 0$

$x = 1 + \sqrt{2}$

$x - 1 = \sqrt{2}$ ← get the root by itself

$(x-1)^2 = (\sqrt{2})^2$

$x^2 - 2x + 1 = 2$

$x^2 - 2x - 1 = 0$

$$P(x) = (x^2 - 8x + 20)(x^2 - 2x - 1)$$

$$= x^4 - 2x^3 - x^2 - 8x^3 + 16x^2 + 8x + 20x^2 - 40x - 20$$

$$\boxed{P(x) = x^4 - 10x^3 + 35x^2 - 32x - 20}$$

$x = 4 \pm 2i \qquad x = 1 \pm \sqrt{2}$

Ex. roots: $x = 2$, $x = \sqrt{2}$, $x = i$

$x = 2$

$x - 2 = 0$

$x = \sqrt{2}$

$x^2 = 2$

$x^2 - 2 = 0$

$x = i$

$x^2 = -1$

$x^2 + 1 = 0$

$$P(x) = (x-2)(x^2-2)(x^2+1)$$

$$= (x^3-2x^2-2x+4)(x^2+1)$$

$$= x^5-2x^4-2x^3+4x^2+x^3-2x^2-2x+4$$

$$= \underline{x^5-2x^4-x^3+2x^2-2x+4}$$

HW p. 276

~~#~~ **36, 40, 46, 48, 55, 60**