$\boxed{\#43.}$ $x^5+3x^4-9x^3-31x^2\overset{0x}{+}36$

$$\begin{array}{r|rrrrr|r} 1 & 1 & 3 & -9 & -31 & 0 & 36 \\ & & 1 & 4 & -5 & -36 & -36 \\ \hline & 1 & 4 & -5 & -36 & \underline{-36} & \boxed{0} \end{array}$$

$\underline{(x-1)}\ (x^4+4x^3-5x^2-36x-36)$

$$\begin{array}{r|rrrr|r} -2 & 1 & 4 & -5 & -36 & -36 \\ & & -2 & -4 & 18 & 36 \\ \hline & 1 & 2 & -9 & -18 & \boxed{0} = f(-2) \end{array}$$

$(x-1)(x+2)(x^3+2x^2-9x-18)$

$$\begin{array}{r|rrr|r} -2 & 1 & 2 & -9 & -18 \\ & & -2 & 0 & 18 \\ \hline & 1 & 0 & -9 & \boxed{0} \end{array}$$

$(x-1)(x+2)^2(x^2-9)$

$\boxed{(x-1)(x+2)^2(x-3)(x+3)}$

zeros (roots)

1, −2, ±3

Descartes' Rule

p. 261 #65

$P(x) = +x^3 - x^2 - x - 3$ 1 positive root

$P(-x) = (-x)^3 - (-x)^2 - (-x) - 3$

$= -x^3 - x^2 + x - 3$

2 or $\underline{\underline{0}}$ negative roots

#67. $2x^6 + 5x^4 - x^3 - 5x - 1$ $\boxed{\text{1 pos. root}}$

$P(-x) = 2(-x)^6 + 5(-x)^4 - (-x)^3 - 5(-x) - 1$

$= 2x^6 + 5x^4 + x^3 + 5x - 1$

$\boxed{\text{1 neg. root}}$

$\boxed{\text{4 non-real roots}}$

#47 $P(x) = x^3 + 4x^2 + 3x - 2$

$-x^3 + 4x^2 - 3x - 2$ 2 or 0 neg. root

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 3 & -2 \\ & & -2 & -4 & 2 \\ \hline & 1 & 2 & -1 & \boxed{0} \end{array} = f(-2)$$

$(x+2)(x^2 + 2x - 1) = 0$

$\boxed{x = -2}$

$x = \dfrac{-2 \pm \sqrt{4 - -4}}{2}$

$\sqrt{8}$

$\sqrt{4 \cdot 2}$

$\sqrt{4} \cdot \sqrt{2}$

$2\sqrt{2}$

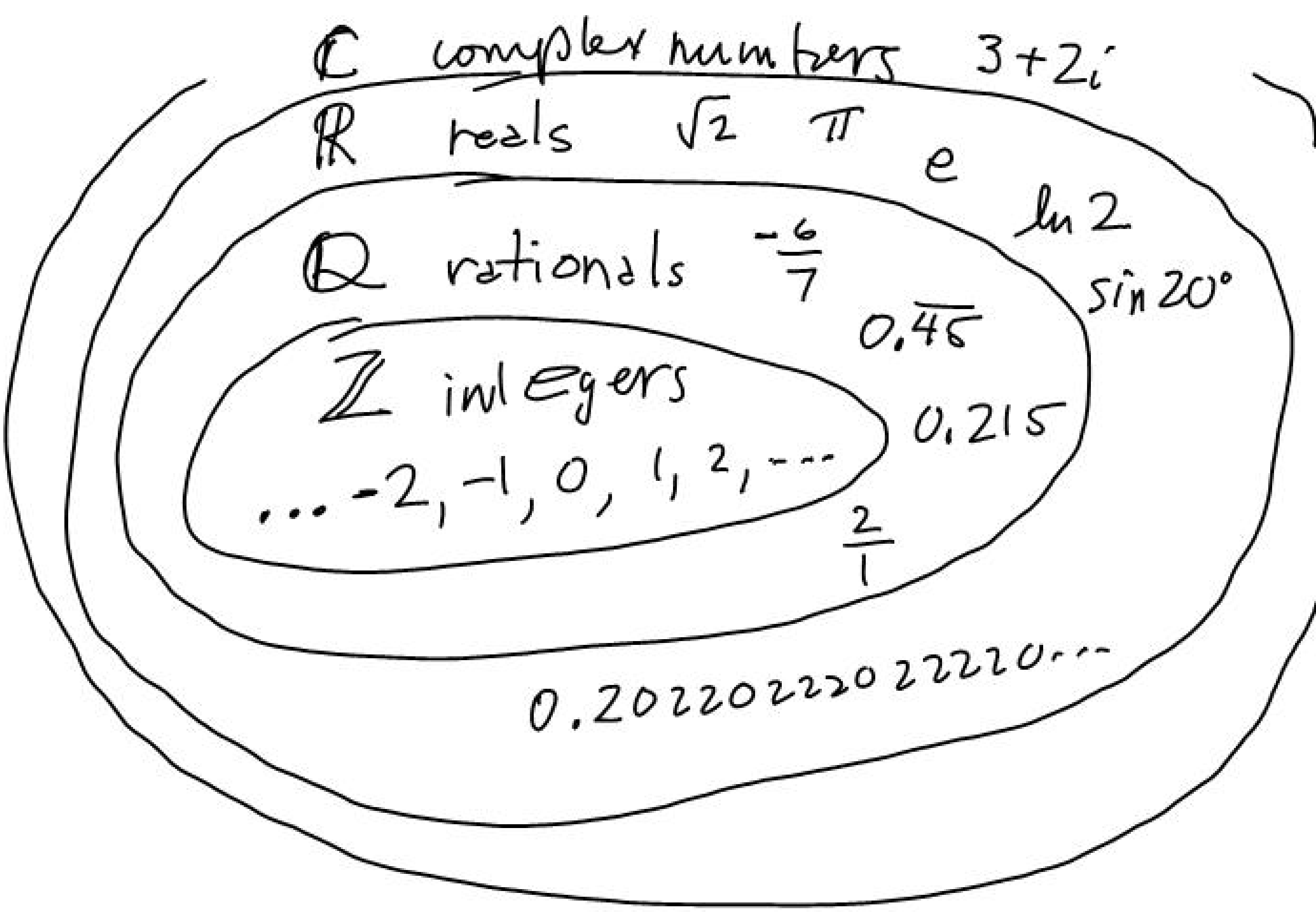
$\boxed{x = -1 \pm \sqrt{2}}$

$x = \dfrac{-2 \pm \cancel{\sqrt{8}}\ 2\sqrt{2}}{2}$

$x = \dfrac{\cancel{2}(-1 \pm \sqrt{2})}{\cancel{2}}$

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



$$\sqrt{-1} = i$$

$$\sqrt{-4} = \sqrt{-1 \cdot 4} = 2i$$

$$\sqrt{-20} = \sqrt{-1 \cdot 4 \cdot 5} = 2\sqrt{5}\, i$$

Add and subtract

$$(\underline{2} + \underline{\underline{5}}i) - (\underline{-1} + \underline{\underline{4}}i) = 3 + i$$

Multiplication

$$(2+5i)(-1+4i)$$

$$= -2 + 8i - 5i + 20\cancel{i^2}(-1)$$

$$= -2 + 3i - 20$$

$$= -22 + 3i$$

$$i^2 = -1$$

$$i^3 = i \cdot i^2 = -i$$

$$i^4 = i^2 \cdot i^2 = 1$$

$$i^5 = i^4 \cdot i = i$$

$$i^{2017} = i^{2016} \cdot i^1$$

$$= i$$

Division

The <u>conjugate</u> of $a+bi$ is $a-bi$

Ex. $$\frac{2+5i}{-1+4i} \cdot \frac{-1-4i}{-1-4i} = \frac{-2-8i-5i-20i^2}{1-16\cancel{i^2}(-1)}$$

$$= \frac{+18-13i}{17}$$

$$= \frac{18}{17} - \frac{13}{17}i$$

square roots

$(a+bi)(a+bi)$

Ex. $\left(\sqrt{3+4i}\right)^2 = (a+bi)^2$

$3+4i = a^2 + 2abi + b^2\cancel{i^2}(-1)$

$3+4i = (a^2-b^2) + (2ab)i$

$$\begin{cases} a^2-b^2 = 3 \\ 2ab = 4 \end{cases} \longrightarrow b = \frac{4}{2a} = \frac{2}{a}$$

$a^2 - \left(\frac{2}{a}\right)^2 = 3$

$a^2\left(a^2 - \frac{4}{a^2}\right) = (3)\,a^2$

$a^4 - 4 = 3a^2$

$a^4 - 3a^2 - 4 = 0$

$(a^2+1)(a^2-4) = 0$

$\cancel{(a^2+1)}(a+2)(a-2)=0$

$a=-2$ or $a=2$

$b=-1$ or $b=1$

$\boxed{-2-i}$ or $\boxed{2+i}$

$\sqrt{3+4i}$

check

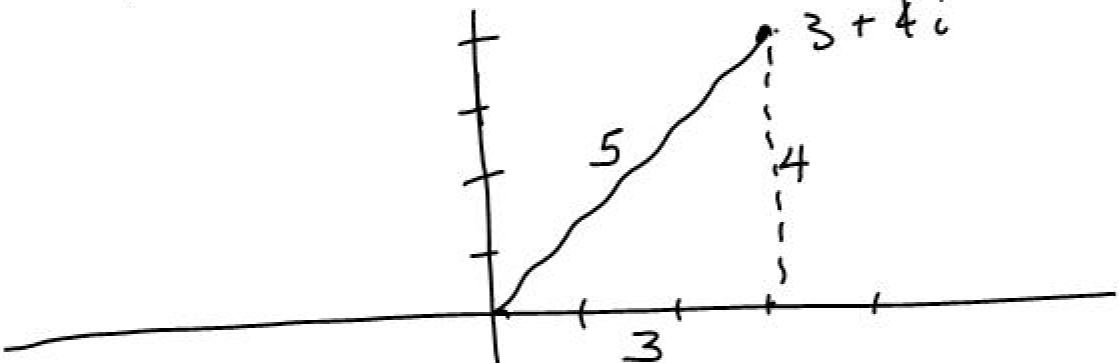
$(2+i)(2+i)$

$4+4i+\cancel{i^2}\,{-1}$

$3+4i$

Absolute value

$$|3+4i| = \sqrt{3^2+4^2} = \sqrt{25} = 5$$



$$\boxed{|a+bi| = \sqrt{a^2+b^2}}$$

HW

p. 261 # 49, 50, 68–70

p. 268 # 16, 20, 25, 27, 29,

34, 36, 37, 43

$$\frac{4+6i}{3i}\cdot\frac{-3i}{-3i}$$

$0+3i$

$0-3i$