Polynomials

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n$$

constant polynomial Ex. $f(x) = 3\sqrt{2}$

linear polynomial Ex. $f(x) = 4 - 5x$

quadratic polynomial Ex. $f(x) = 1 - 2x + 5x^2$

cubic polynomial Ex. $f(x) = 4 - x + 3x^2 + 4x^3$

quartic (4th degree)

quintic (5th degree)

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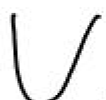
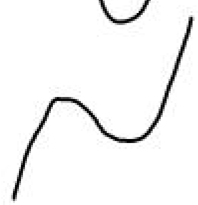
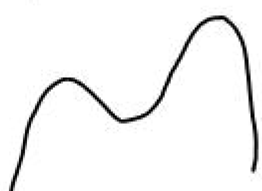
quadratic  1 turn

cubic  2 turns or no turns

quartic  3 turns or 1 turn.

quintic  4 turns, 2, or 0

Quadratic Polynomials

Finding the vertex by completing the square

Ex. $f(x) = x^2 - 6x + 10$

$$= (x^2 - 6x \underline{+9}) + 10 \underline{-9}$$

$\left[\frac{1}{2}(-6)\right]^2$

$$\boxed{f(x) = (x-3)^2 + 1}$$

vertex $(3,1)$

Where is the minimum? $\underline{x=3}$

What is the minimum? $\underline{y=1}$

What is the range? $\underline{[1, \infty)}$

Ex. $f(x) = 3x^2 - 9x + 1$

$\frac{4}{4} - \frac{27}{4}$

$= 3\left(x^2 - 3x + \frac{9}{4}\right) + 1 - \frac{27}{4}$

$\left[\frac{1}{2}(-3)\right]^2$

$\left(-\frac{3}{2}\right)^2$

$= 3\left(x - \frac{3}{2}\right)^2 - \frac{23}{4}$

vertex $\left(\frac{3}{2}, -\frac{23}{4}\right)$

Ex. $f(x) = 4 + x - 5x^2$

$\frac{-5}{100}$

$= -5\left(x^2 - \frac{1}{5}x + \frac{1}{100}\right) + 4 + \frac{1}{20}$

$\left[\frac{1}{2}\left(-\frac{1}{5}\right)\right]^2$

$\left(-\frac{1}{10}\right)^2$

$= -5\left(x - \frac{1}{10}\right)^2 + \frac{81}{20}$

vertex $\left(\frac{1}{10}, \frac{81}{20}\right)$

max. value

Ex. The height of a ball thrown upward is $y = 5 + 25t - 16t^2$; y in feet and t in secs. Find the max height of the ball.

$$y = -16\left(t^2 - \frac{25}{16}t + \frac{625}{1024}\right) + 5 + \frac{625}{64}$$

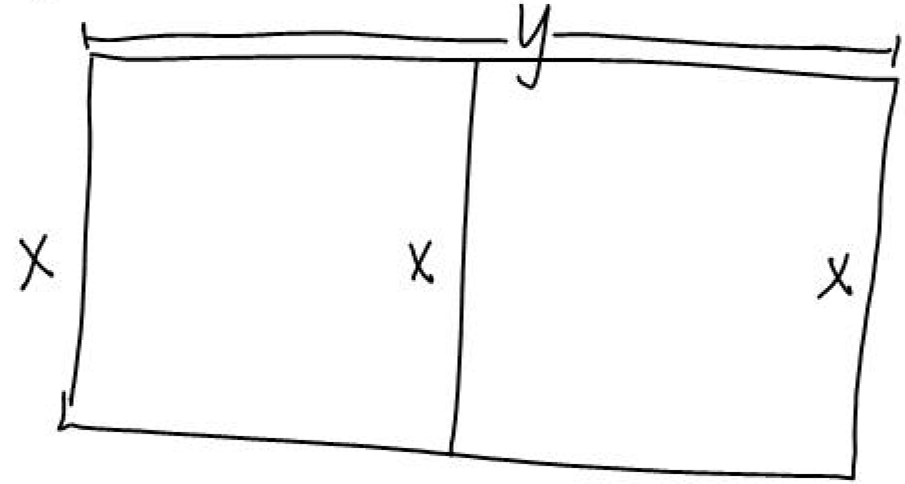
$$\left[\frac{1}{2}\left(\frac{-25}{16}\right)\right]^2$$

$$\boxed{y = -16\left(t - \frac{25}{32}\right)^2 + \frac{945}{64}}$$

$$\left(\frac{-25}{32}\right)^2$$

Max. height is $\frac{945}{64}$ ft or ≈ 14.8 ft

Ex. A rancher wants to build a corral that is rectangular and divided into 2 sections.



He has 2000 feet of fence. Find the largest possible area for the corral.

Area $= xy$ $\qquad$ $3x + 2y = 2000$

$$y = \frac{2000 - 3x}{2}$$

$$A(x) = x\left(\frac{2000 - 3x}{2}\right)$$

$$A(x) = 1000x - \frac{3}{2}x^2$$

$$\cancel{-\tfrac{3}{2}}\,Z = (1000)\left(-\tfrac{2}{3}\right)$$

$$A(x) = -\frac{3}{2}\left(x^2 - \frac{2000}{3}x + \frac{1000000}{9}\right) + \frac{500000}{3}$$

$$\left[\frac{1}{2}\left(-\frac{2000}{3}\right)\right]^2$$

$$\left(\frac{-1000}{3}\right)^2$$

$$-\frac{\cancel{3}}{\cancel{2}} \cdot \frac{\overset{500000}{\cancel{1000000}}}{\cancel{9}\,3}$$

maximum area is $\frac{500\,000}{3}$ ft^2

or 166,667 ft^2