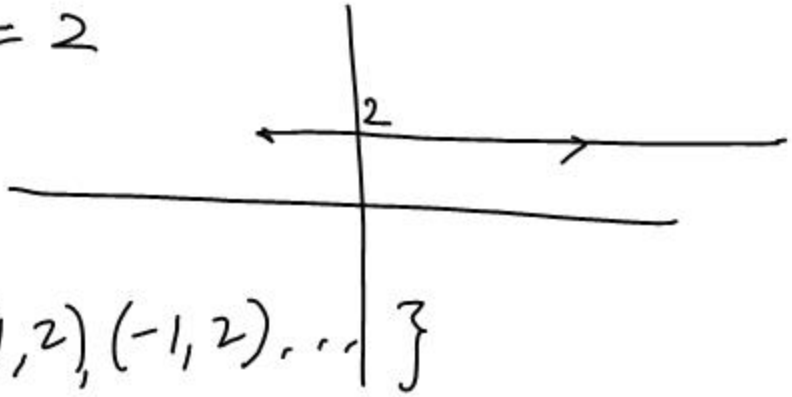
Functions

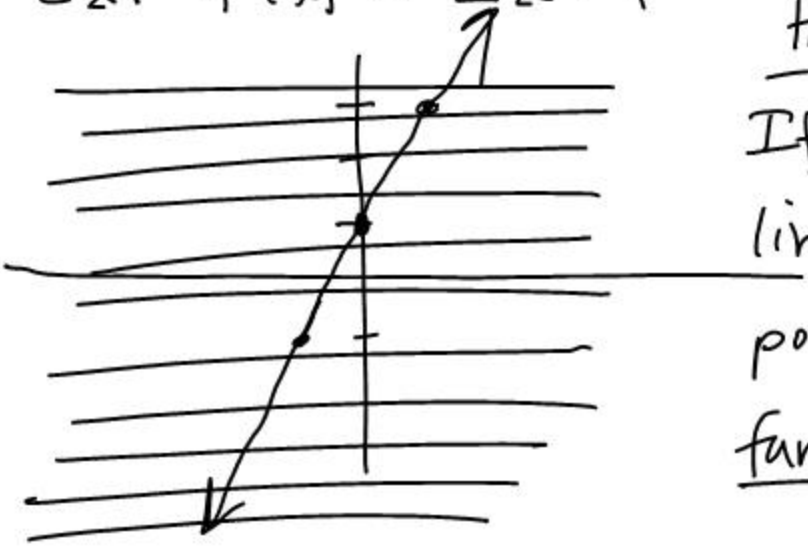
$y = 2$



$\{(0,2), (1,2), (-1,2), \ldots\}$

If no y-values are repeated in a function, then it is a one-to-one function.

Ex. $f(x) = 2x + 1$

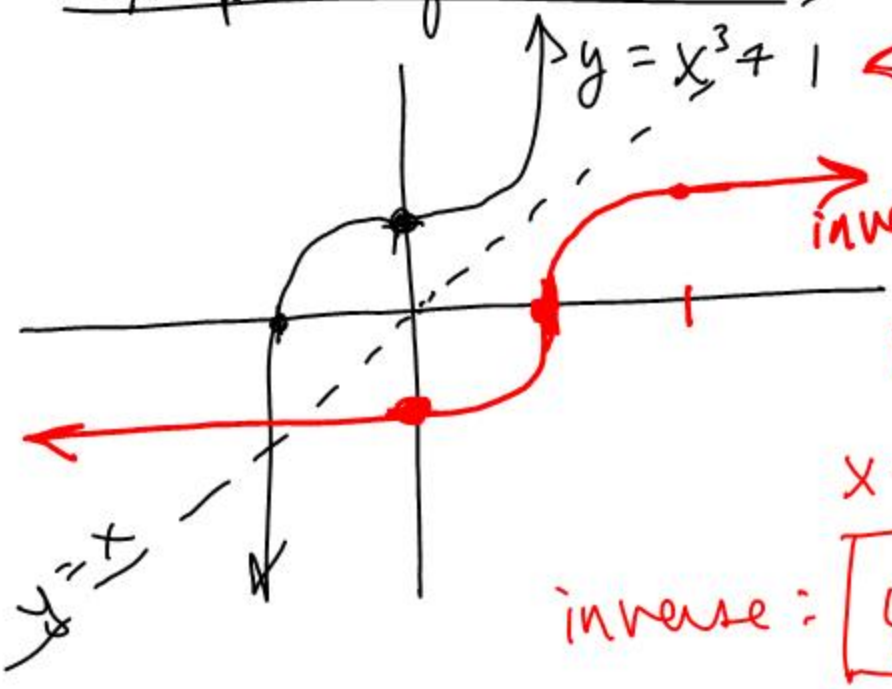


Horizontal Line Test

If no horizontal line hits more than 1 point on the graph of a function, then it's 1-1.

1-1 functions have inverses. An inverse function is what you get when you swap all the x's and y's.

Graphs of inverses



Ex. Find the inverse function for $f(x) = \dfrac{x+2}{x-1}$

Inverse: $x = \dfrac{y+2}{y-1}$ (swap x and y)

$yx - x = y + 2$ (solve for y)

$yx - y = 2 + x$

$y(x-1) = 2 + x$

$y = \dfrac{2+x}{x-1}$

$$\boxed{f^{-1}(x) = \frac{2+x}{x-1}}$$

Find $f^{-1}(x)$.

$$x = \frac{2y+5}{3y-4}$$

$$3xy - 4x = 2y + 5$$

$$3xy - 2y = 5 + 4x$$

$$y(3x-2) = 5 + 4x$$

$$\boxed{y = f^{-1}(x) = \frac{5+4x}{3x-2}}$$

check with 1 ordered pair. $f(1) = \frac{7}{-1} = -7$

$(1, -7)$

$$f^{-1}(-7) = \frac{5+4(-7)}{3(-7)-2}$$

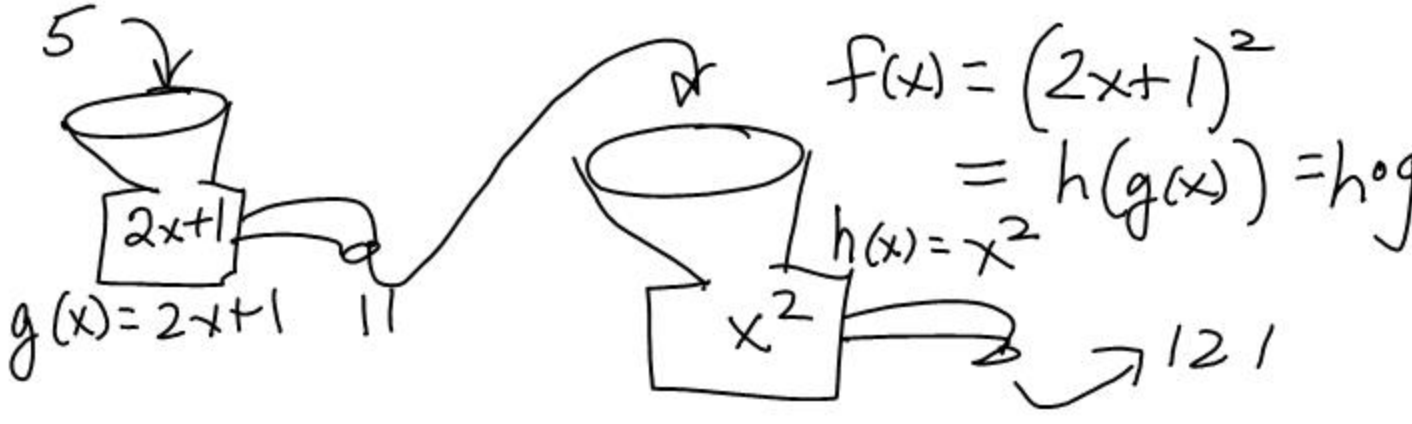
$$= \frac{-23}{-23} = +1$$

$(-7, 1)$

$y = e^x$ $\quad y = \sin x$

$y = \ln x$ $\quad y = \sin^{-1} x$

Composition of Functions



$f(x) = (2x+1)^2$

$= h(g(x)) = h \circ g$

Ex. $f(x) = \sqrt{x}$

$g(x) = x^2 + 4$

$f(g(x)) = f(x^2+4) = \sqrt{x^2+4}$

$g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 + 4$

$= (g \circ f)(x)$ $\qquad = x + 4$

Ex $f(x) = \dfrac{2x+5}{3x-4}$ $\qquad f^{-1}(x) = \dfrac{5+4x}{3x-2}$

$$f(f^{-1}(x)) = f\left(\frac{5+4x}{3x-2}\right) = \frac{2\left(\frac{5+4x}{3x-2}\right)+5}{3\left(\frac{5+4x}{3x-2}\right)-4}$$

$$= \frac{\frac{10+8x}{3x-2} + \frac{15x-10}{3x-2}}{\frac{15+12x}{3x-2} - \frac{12x-8}{3x-2}} = \frac{\frac{23x}{3x-2}}{\frac{23}{3x-2}}$$

$$= \frac{23x}{3x-2} \cdot \frac{3x-2}{23} = x = f^{-1}(f(x))$$

EX. $f(x) = x^2$ and $g(x) = \sqrt{x}$

$(4, 16)$ $(16, 4)$

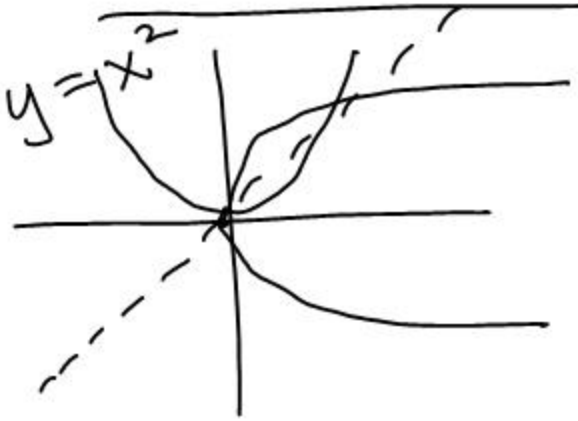
Are these inverses?

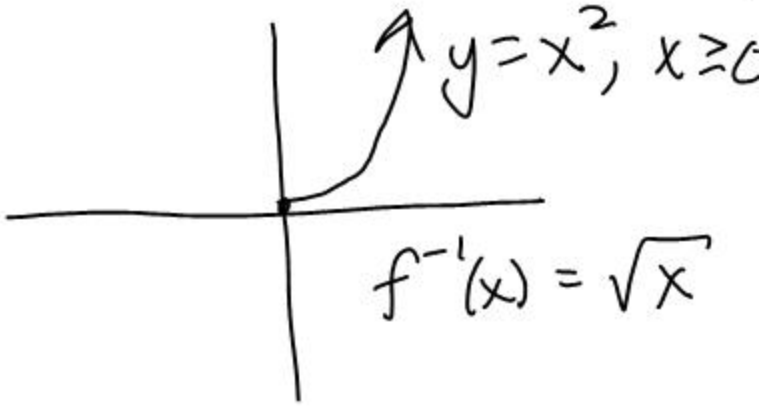
$$f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$$

$$g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$$

$$\sqrt{(-5)^2} = \sqrt{25} = 5$$

$|-5|$





Undoing Compositions

$$F(x) = |x^3 - 1| = f(g(x))$$

$$f(x) = |x|$$

$$g(x) = x^3 - 1$$

HW. p. 197 # 33, 39, 44; 49, 51

$f \circ g = f(g(x))$

$g \circ f = g(f(x))$

$f \circ f = f(f(x))$

p. 205

37, 41, 47*, 50*

Friday: Test One Review

Tuesday 9-12 TEST ONE