☒ Review

(i) $2a^2 + 3a + (3a^2 + 2a) + (2a^2 + a) = 1$

$7a^2 + 6a - 1 = 0$

$(7a - 1)(a + 1)$

$\boxed{a = 1/7}$ or $\cancel{a = -1}$

Hence:

X	1	2	3	4
$p(X=x)$	$\frac{2}{49}$	$\frac{3}{7} = \frac{21}{49}$	$\frac{17}{49}$	$\frac{9}{49}$

(ii) $E(X) = \frac{2}{49}(1) + \frac{21}{49}(2) + \frac{17}{49}(3) + \frac{9}{49}(4)$

$= \frac{131}{49}$ or 2.67

mode = 2

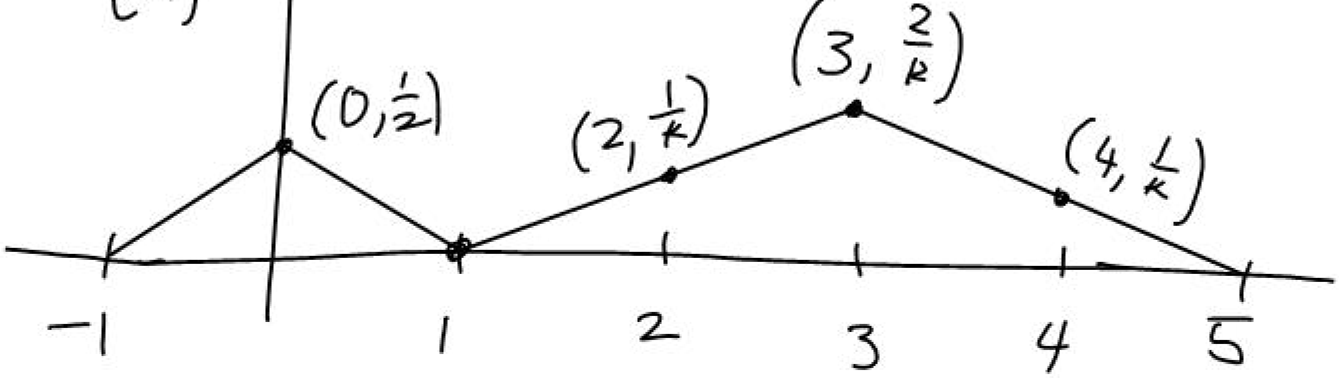
median falls between 2 and 3

because $P(X \le 2) < 0.5$ and $P(X \le 3) > 0.5$

median = 2.5

2)

(a)



$$\frac{1}{2}(2)\left(\frac{1}{2}\right) + \frac{1}{2}(4)\left(\frac{2}{k}\right) = 1$$

$$\frac{4}{k} = \frac{1}{2}$$

$$\underline{k = 8}$$

(b) median = 1 as it splits the area into 2 equal sections

mode = 0 (highest point on the graph)

(c) $P(0 \le X \le 3 \mid X \ge 1) = \dfrac{P(1 \le X \le 3)}{P(X \ge 1)}$

$$= \frac{\cancel{\frac{1}{2}}\cancel{(2)}\left(\frac{1}{4}\right)}{\frac{1}{2}} = \frac{1}{2}$$

(3) X = number of graduates in a sample of 5

$X \sim B(5, 4/5)$

(a) $P(X=0) = \binom{5}{0}\left(\frac{4}{5}\right)^0\left(\frac{1}{5}\right)^5 = \frac{1}{5^5}$

(b) $P(X=5) = \binom{5}{5}\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)^0 = \frac{4^5}{5^5}$

(c) $P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - P(X=0) - P(X=1)$

$= 1 - \frac{1}{5^5} - 5 \cdot \left(\frac{4}{5}\right)^1\left(\frac{1}{5}\right)^4$

$= 1 - \frac{1}{5^5} - \frac{20}{5^5}$

$= \frac{5^5 - 21}{5^5}$

(4) X = number correct on 5 guesses

$X \sim B(5, 1/2)$

(a) $P(X=5) = \binom{5}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 = \frac{1}{32}$

$E(X) = n \cdot p$

(b) $E(X) = 5 \cdot \frac{1}{2} = 2.5$

So we expect he gets 7.5 correct

This gives him $2(7.5) - 1(2.5) = 12.5$ pts

Answering only 5 : $2(5) - 0 = 10$

So he can expect to get 2.5 more pts. by guessing.

(5) $E(T^2) = 6$ and $\sigma^2 = E(T^2) - (E(T))^2$

so, $6 - [E(T)]^2 = \sigma^2$

Poisson
$P(m)$
$m = E(X)$
$m = \sigma^2$

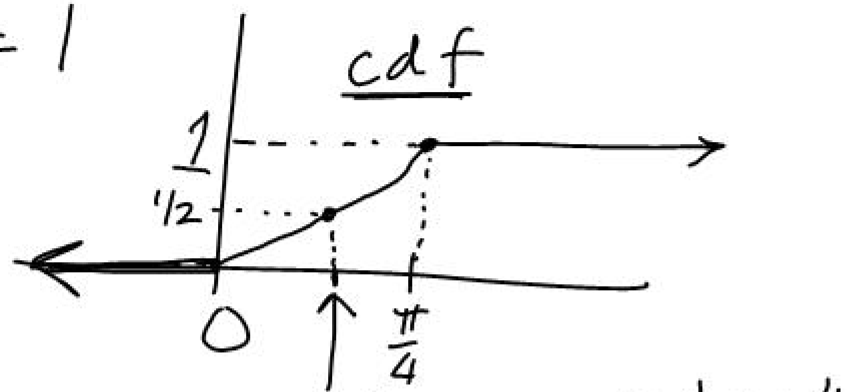
$6 - m^2 = m$

$m^2 + m - 6 = 0$

$(m+3)(m-2) = 0$

$\cancel{m = -3}$ or $\boxed{m = 2}$

⑥ (a) $a = 1$



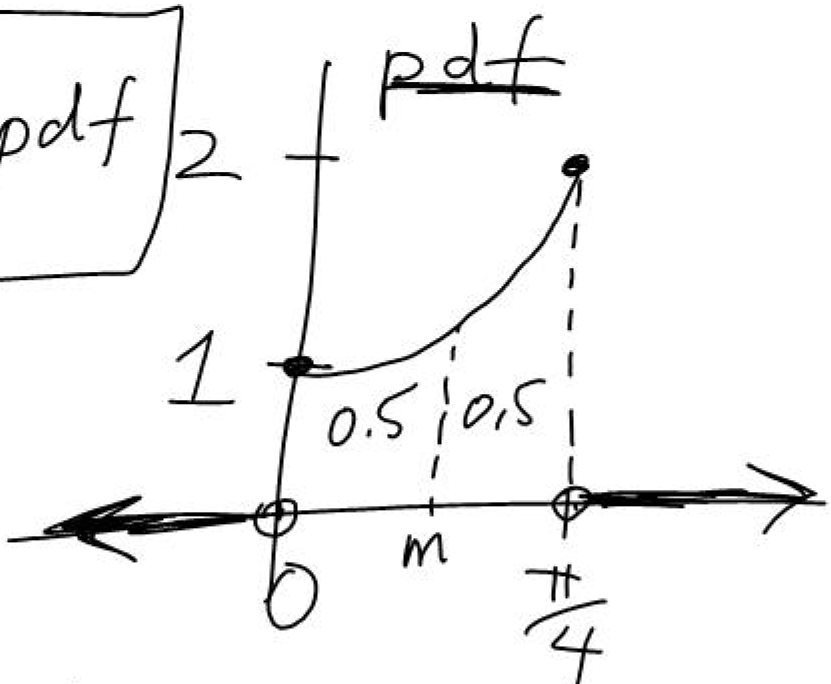
(b) $\tan x = \frac{1}{2}$

$x = \arctan\left(\frac{1}{2}\right)$

(c) $f(x) = \frac{d}{dx}\left[F(x)\right] = \begin{cases} \sec^2 x, & 0 \le x \le \frac{\pi}{4} \\ 0, & \text{elsewhere} \end{cases}$

(d) $P\left(X \le \frac{\pi}{6}\right) = F\left(\frac{\pi}{6}\right) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$$\boxed{F = \text{cdf} = \int \text{pdf}}$$



$\int_0^m \sec^2 x \, dx = \frac{1}{2}$

$\left[\tan x\right]_0^m = \frac{1}{2}$

$\tan m - \cancel{\tan 0}^{\,0} = \frac{1}{2} \longrightarrow m = \tan^{-1}\frac{1}{2}$

another look (d) using the pdf:

$$P\left(X \le \frac{\pi}{6}\right) = \int_0^{\pi/6} \sec^2 x \, dx$$

$$= \left[\tan x\right]_0^{\pi/6}$$

$$= \tan\frac{\pi}{6} - \cancel{\tan 0}^{\,0} = \frac{1}{\sqrt{3}}$$

Review

$\boxed{\#3}$ pdf: $f(x) = \begin{cases} \dfrac{k}{2+x^2}, & 0 \le x \le \sqrt{2} \\ 0, & \text{elsewhere} \end{cases}$

(a) $\displaystyle\int_0^{\sqrt{2}} \frac{k}{2+x^2}\, dx = 1 \Rightarrow k = 1.80$

(b) $\displaystyle P\left(X \le \tfrac{1}{2}\right) = \int_0^{1/2} \frac{1.800}{2+x^2}\, dx = 0.433$

(c) $$E(X) = \int_0^{\sqrt{2}} \frac{1.800x}{2+x^2}\,dx = 0.624$$

OR by hand:

$$\frac{1.8}{2}\int_0^{\sqrt{2}} \frac{2x}{2+x^2}\,dx = 0.9\int_2^4 \frac{du}{u}$$

$$u = 2 + x^2$$

$$du = 2x\,dx$$

$$= 0.9\Big[\ln u\Big]_2^4$$

$$u(0) = 2 + 0^2 = 2$$

$$u(\sqrt{2}) = 2 + \sqrt{2}^2 = 4$$

$$= 0.9(\ln 4 - \ln 2)$$

$$= 0.9 \ln 2$$

$$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$$

(d) Find the variance of the distribution

$$E(X^2) = 1.8\int_0^{\sqrt{2}} \frac{x^2}{2+x^2}\,dx = 0.54629$$