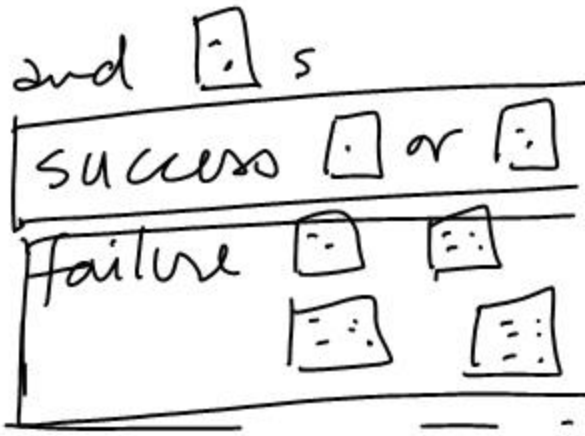
The Binomial Distribution

- fixed number of trials
- fixed probability success on each trial
- Each trial is either a success or a failure) 2 outcomes

Ex. Roll 5 dice.

X = number of ⚀s and ⚁s

5 trials, $p = \frac{1}{3}$,



X	0	1	2	3	4	5
$P(X)$						

Probability Distribution

X	0	1	2	3	4	5
P(X)	$\frac{32}{243}$	$\frac{80}{243}$	$\frac{80}{243}$	$\frac{40}{243}$	$\frac{10}{243}$	$\frac{1}{243}$

$$P(X=0) = \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}$$

$$= \left(\frac{2}{3}\right)^5 = \frac{32}{243}$$

$$P(X=1) = 5\cdot\left(\frac{1}{3}\right)\cdot\left(\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\right) = \frac{80}{243}$$

$$= \binom{5}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^4$$

n = number trials

p = probability of success

$q = 1 - p$ = probability of failure

$$P(X=x) = \binom{n}{x}(p)^x(q)^{n-x}$$

$$P(X=2) = \binom{5}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3$$

$$= 10 \cdot \frac{8}{243}$$

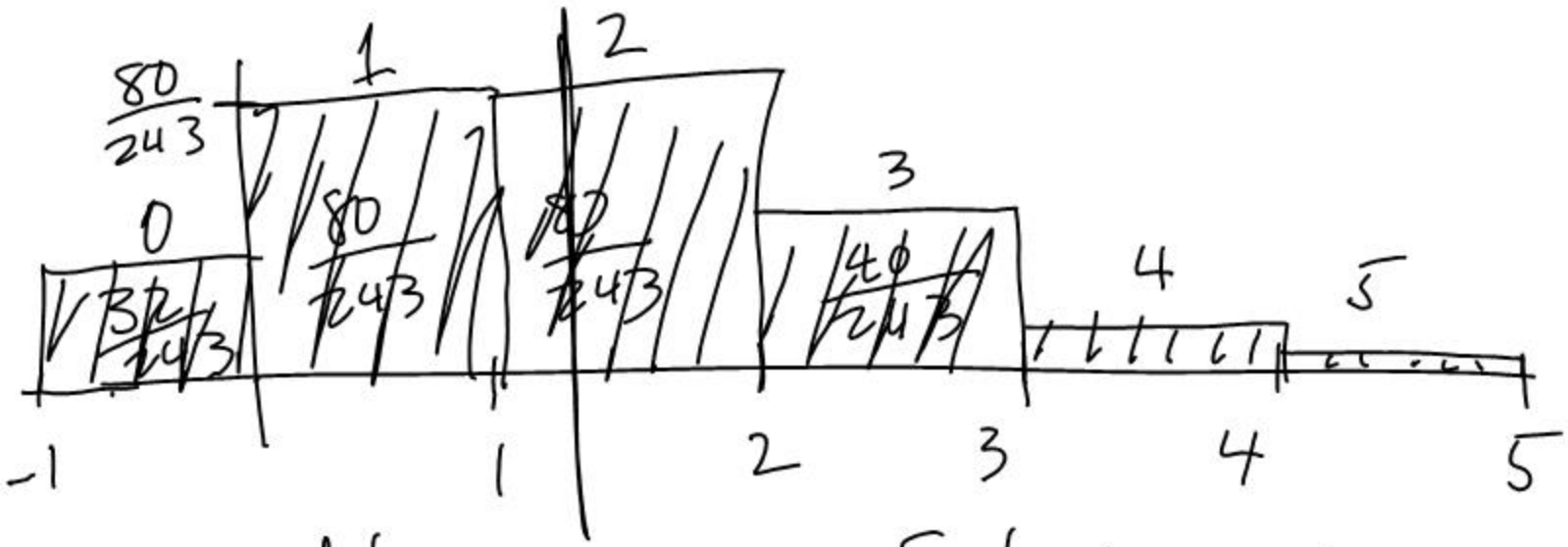
$$P(X=3) = \binom{5}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2$$

$$= 10 \cdot \frac{4}{243}$$

$$P(X=4) = \binom{5}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1$$

$$= 5 \cdot \frac{2}{243}$$

$$P(X=5) = \binom{5}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^0 = \frac{1}{243}$$



pdf $f(x) = \begin{cases} \frac{1}{x^2}, & x \geq 1 \\ 0, & \text{elsewhere} \end{cases}$

$E(X) = \int_a^b x \cdot f(x)$

σ^2, σ

median

mode

$E(X^2) = \int x^2 \cdot f(x)$

For discrete distributions:

$$E(X) = \sum x \cdot P(x)$$

For the 5 dice distribution: $\boxed{E(X) = \frac{5}{3}}$

$$E(X) = 0\left(\frac{32}{243}\right) + 1\left(\frac{80}{243}\right) + 2\left(\frac{80}{243}\right) + 3\left(\frac{40}{243}\right) + 4\left(\frac{10}{243}\right) + 5\left(\frac{1}{243}\right)$$

$$\sigma^2 = E(X^2) - [E(X)]^2$$

$$E(X^2) = 0^2\left(\frac{32}{243}\right) + 1^2\left(\frac{80}{243}\right) + 2^2\left(\frac{80}{243}\right)$$

$$+ 3^2\left(\frac{40}{243}\right) + 4^2\left(\frac{10}{243}\right) + 5^2\left(\frac{1}{243}\right)$$

$$E(X^2) = \frac{35}{9}$$

$$\sigma^2 = \frac{35}{9} - \left(\frac{5}{3}\right)^2 = \frac{10}{9}$$

Standard deviation: $\sigma = \frac{\sqrt{10}}{3} \approx 1.05$

mode: 1 and 2

median: 2

HW Roll 3 dice

X = number of 

x	0	1	2	3
$P(x)$				

$E(x) =$

$\sigma^2 =$