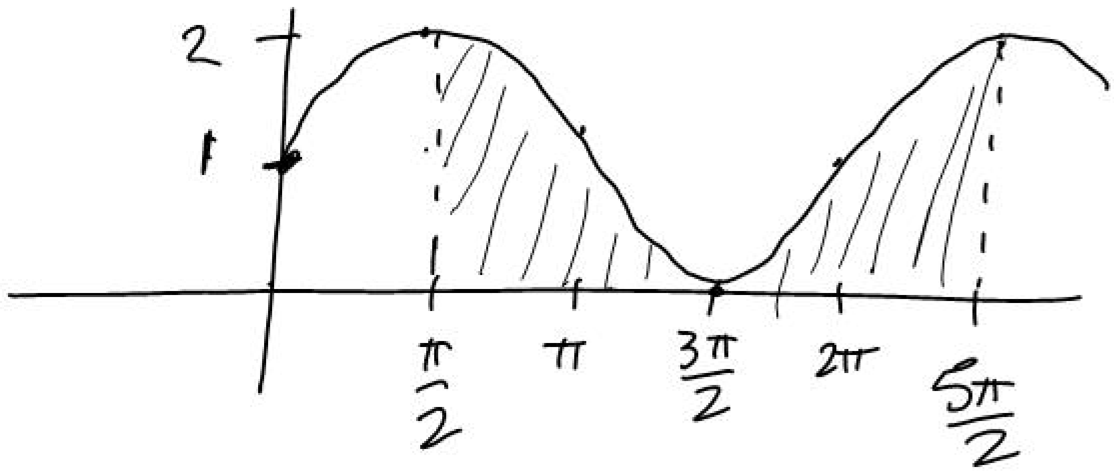
#8

$$f(x) = \alpha(1+\sin x), \quad \frac{\pi}{2} \le x \le \frac{5\pi}{2}$$

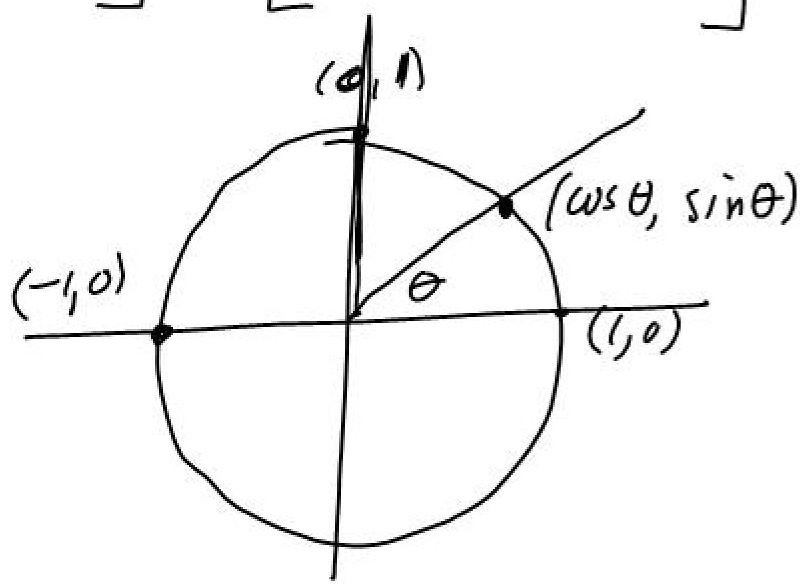


$$\int_{\pi/2}^{5\pi/2} (1+\sin x)\,dx = \Big[x - \cos x\Big]_{\pi/2}^{5\pi/2}$$

$$= \left[\frac{5\pi}{2} - \cancel{\cos\frac{5\pi}{2}}\right] - \left[\frac{\pi}{2} - \cancel{\cos\frac{\pi}{2}}^{\,0}\right]$$

$$= 2\pi$$

Let $\alpha = \frac{1}{2\pi}$



$$P(X < \pi) = \frac{1}{2\pi}\int_{\pi/2}^{\pi} (1 + \sin x)\, dx$$

$$= \frac{1}{2\pi}\Big[x - \cos x\Big]_{\pi/2}^{\pi}$$

$$= \frac{1}{2\pi}\left[(\pi - (-1)) - \left(\frac{\pi}{2} - 0\right)\right] =$$

$$= \frac{1}{2\pi}\left[\frac{\pi}{2} + 1\right] = \frac{1}{4} + \frac{1}{2\pi} \approx 0.409$$

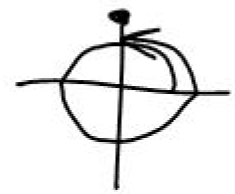
$$P(X < 2\pi) = \frac{1}{2\pi}\left[(2\pi - (-1)) - \left(\frac{\pi}{2} - 0\right)\right]$$

$$= \frac{1}{2\pi}\left[\frac{3\pi}{2} + 1\right] = \frac{3}{4} + \frac{1}{2\pi}$$

$$F(x) = \frac{1}{2\pi}\int_{\pi/2}^{x} (1+\sin t)\,dt$$

$$= \frac{1}{2\pi}\left[t - \cos t\right]_{\pi/2}^{x}$$

$$= \frac{1}{2\pi}\left[\left(x - \cos x\right) - \left(\frac{\pi}{2}\right)\right]$$



check

$$F\left(\frac{5\pi}{2}\right) = \frac{1}{2\pi}\left[\left(\frac{5\pi}{2} - \overset{0}{\cancel{\cos\frac{5\pi}{2}}}\right) - \frac{\pi}{2}\right] = 1$$

CDF

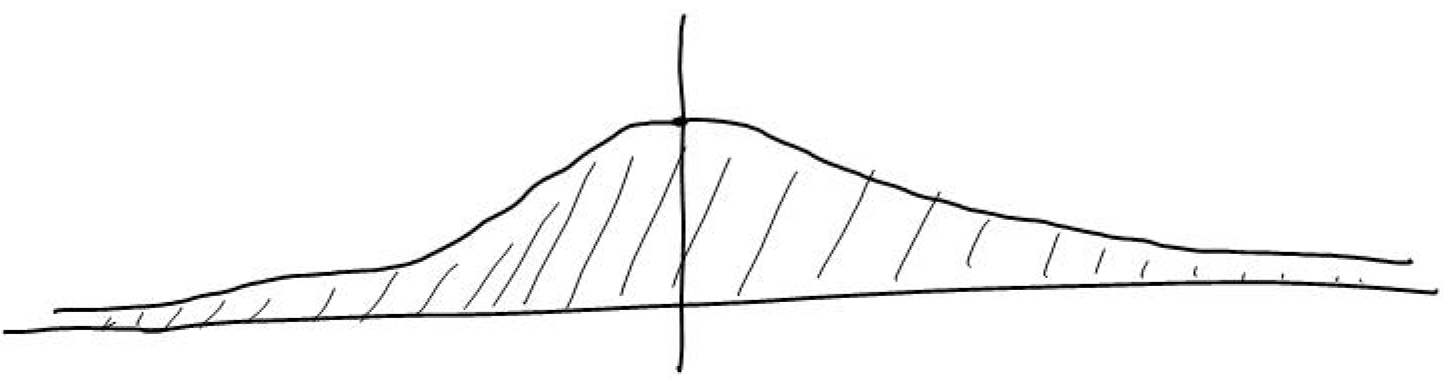
$$F(x) = \begin{cases} 0, & x < \pi/2 \\ \frac{1}{2\pi}\left[x - \cos x - \pi/2\right], & \pi/2 \le x \le \frac{5\pi}{2} \\ 1, & x > 5\pi/2 \end{cases}$$

$$\cancel{\tfrac{1}{2\pi}}\left[x-\cos x-\pi/2\right] = 0.75 = \frac{3}{4}\cdot 2\pi$$

$$x-\cos x-\pi/2 = \frac{3\pi}{2}$$

$$x-\cos x = 2\pi$$

Recall pdf: $f(x) = \dfrac{1}{\pi(x^2+1)}, \quad -\infty < x < \infty$



The Normal Distribution

pdf: $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, $-\infty < x < \infty$

standard dev.

$\approx 68\%$ of data is within $1\ \sigma$ of μ

$\approx 95\%$ " " " " $2\ \sigma$ of μ

mean

$\approx 99.7\%$ " " " " $3\ \sigma$ of μ

Ex. Rutabagas have an average weight of 2 pounds. The variance of the weights is $4\ oz^2$.

A rutabaga is selected at random.

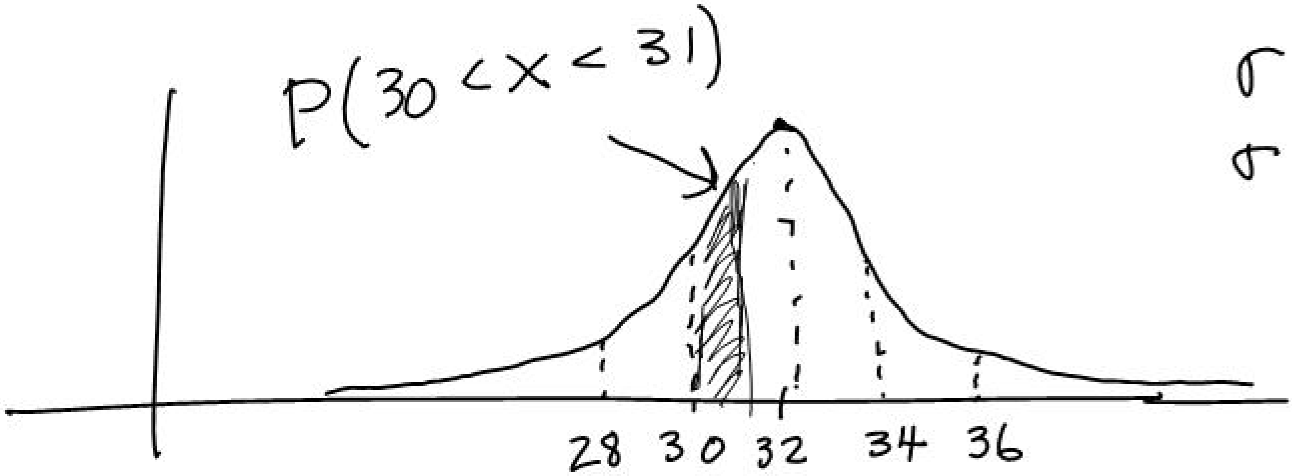
Recall:

measures of spread

σ^2 = variance

σ = standard deviation

Find $P(30 < x < 31\ oz)$



$\sigma^2 = 4\ oz^2$

$\sigma = 2\ oz$

$P(\underline{30} < X < 31) = P(-1 < Z < -\frac{1}{2}) = 0.150$

1σ below μ

$\frac{30-32}{2}$

$\frac{1}{2}\sigma$ below μ

number of standard dev. above or below the mean

- Find $P(X > 33.7\ oz)$

$$\boxed{Z = \frac{X - \mu}{\sigma}}$$

$$z = \frac{33.7 - 32}{2} = 0.85$$

$= P(Z > 0.85) = 0.198$

normalcdf(0.85, 9) $\quad 9 \approx \infty$