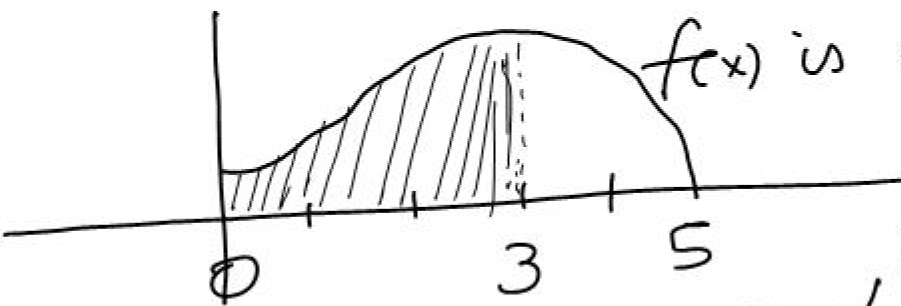
Cumulative Distribution functions (cdfs)



$$P(X \le 3) = \int_0^3 f(x)\,dx$$

pdf

The cdf is $F(x) = \int_0^x f(t)\,dt$.

Ex. Find the cdf for the pdf

$$f(x) = \begin{cases} \frac{3}{8}x^2, & 0 \le x \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

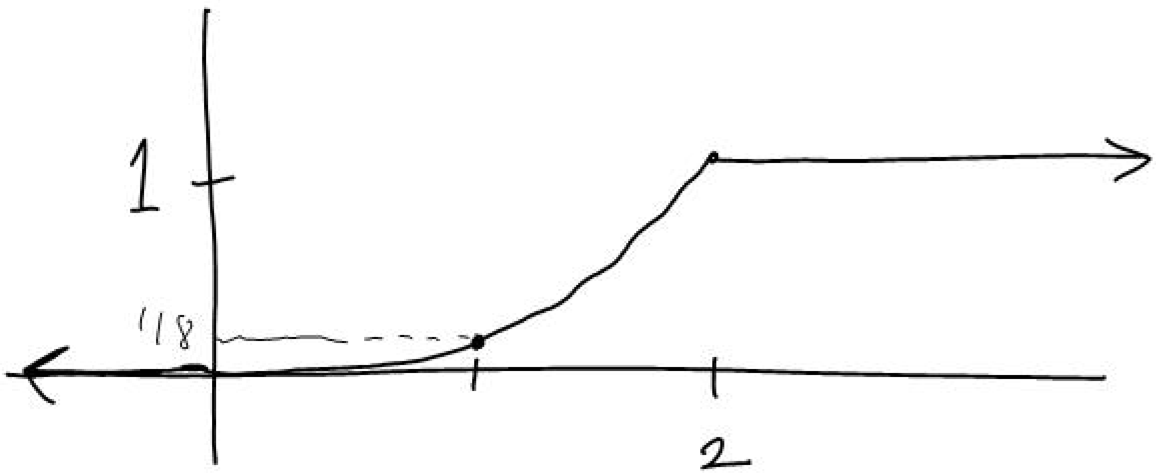
$$P(X \le x) = \int_0^x \frac{3}{8}t^2\,dt = \left[\frac{1}{8}t^3\right]_0^x$$

$$\boxed{F(x) = \frac{1}{8}x^3}$$

cumulative distribution function

$0 \le x \le 2$

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{8}x^3, & 0 \le x \le 2 \\ 1, & x > 2 \end{cases}$$



$$P(X \le 1) = F(1) = \frac{1}{8}$$

$$\text{Ex. } P\left(X > \tfrac{1}{2}\right) = 1 - F\left(\tfrac{1}{2}\right) = \frac{63}{64}$$

Ex. $f(y) = \begin{cases} \lambda(y+3), & -3 \le y \le 3 \\ 0, & \text{otherwise} \end{cases}$

pdf

(a) Find λ

$$\lambda \int_{-3}^{3} (y+3)\, dy = 1$$

$$\lambda \left[\frac{1}{2}y^2 + 3y \right]_{-3}^{3} = 1$$

$$\lambda \left[\left(\tfrac{9}{2} + 9\right) - \left(\tfrac{9}{2} - 9\right) \right] = 1$$

$$18\lambda = 1$$

$$\lambda = \frac{1}{18}$$

$$\boxed{f(y) = \begin{cases} \frac{1}{18}(y+3), & -3 \le y \le 3 \\ 0, & \text{elsewhere} \end{cases}}$$

$-\frac{1}{4}$

$\frac{1}{4} + -\frac{1}{2}$

(b) Find $F(y)$, the cdf for $f(x)$.

$$F(y) = \int_{-3}^{y} \frac{1}{18}(t+3)\, dt = \left[\frac{1}{36}t^2 + \frac{1}{6}t \right]_{-3}^{y}$$

$$= \frac{1}{36}y^2 + \frac{1}{6}y + \frac{1}{4}$$

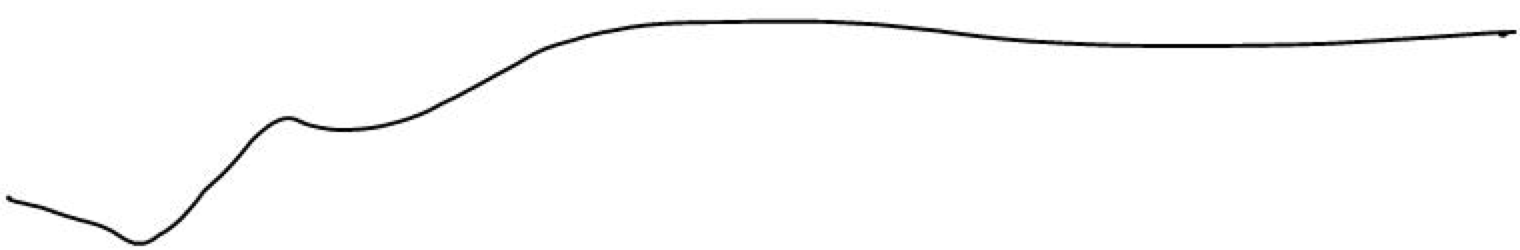
$$\underline{\frac{1}{36}y^2 + \frac{1}{6}y + \frac{1}{4}}$$

$$F(3) = \frac{1}{36}\cdot 9 + \frac{1}{6}\cdot 3 + \frac{1}{4}$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{4}$$

$$F(-3) = \frac{1}{36}\cdot 9 + \frac{1}{6}(-3) + \frac{1}{4}$$

$$= \frac{1}{4} - \frac{1}{2} + \frac{1}{4} \checkmark$$



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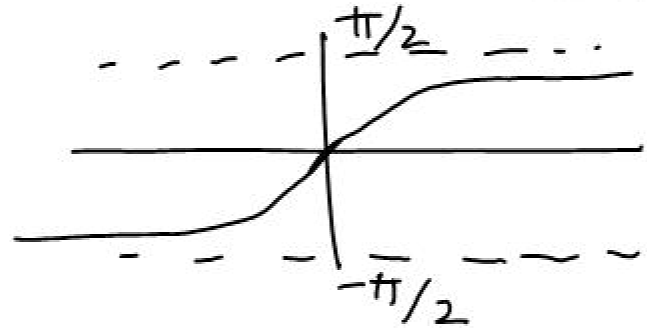
$*$ $$f(x) = \frac{1}{\pi(x^2+1)}$$



$$\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C$$