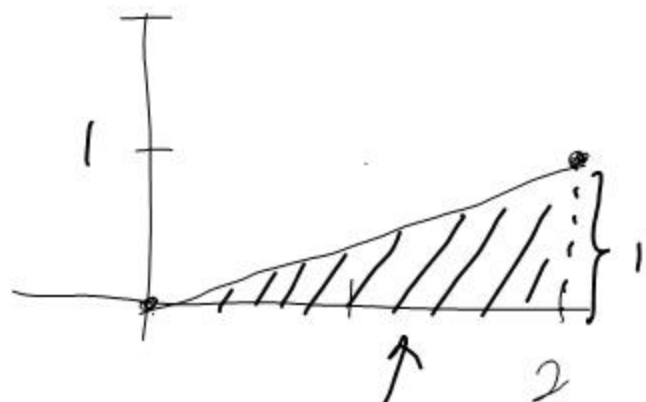
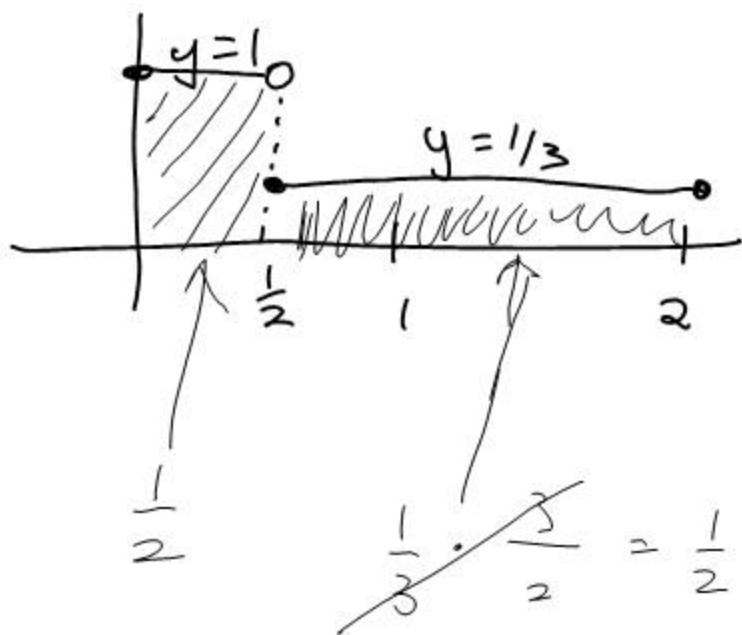


10 I

#2



$$\text{area} = \frac{1}{2}(2)(1) = 1$$



#3

$$(a) E(X) = \int_0^1 x \cdot 6x(1-x) dx$$

$$= \int_0^1 (6x^2 - 6x^3) dx$$

$$= \left[2x^3 - \frac{3}{2}x^4 \right]_0^1 = 2 - \frac{3}{2} = \frac{1}{2}$$

$$\int_0^2 \frac{1}{2} x dx = \left[\frac{1}{4} x^2 \right]_0^2$$

$$= \frac{1}{4}(2)^2 - \frac{1}{4}(0)^2$$

$$= 1$$

$$(b) E(X^2) = \int_0^1 x^2 \cdot 6x(1-x) dx$$

$$= \int_0^1 (6x^3 - 6x^4) dx$$

$$= \left[\frac{3}{2}x^4 - \frac{6}{5}x^5 \right]_0^1 = \frac{3}{2} - \frac{6}{5}$$
$$= \frac{15}{10} - \frac{12}{10}$$

$$E(X^2) = \frac{3}{10}$$

$$\sigma^2 = E(X^2) - [E(X)]^2$$

$$= \frac{3}{10} - \left[\frac{1}{2} \right]^2$$

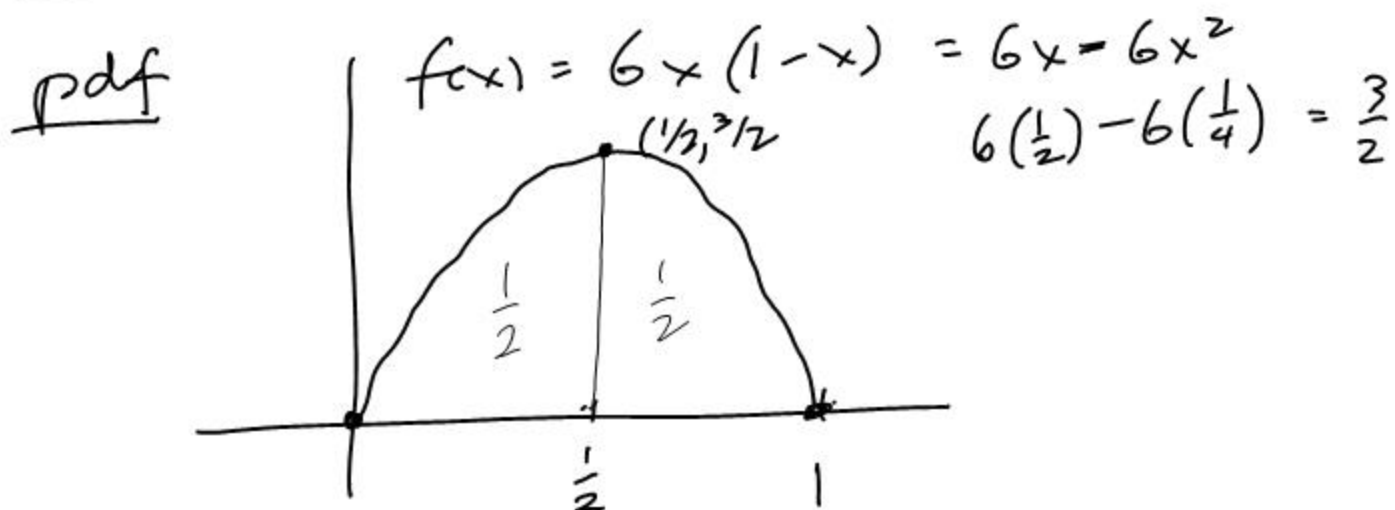
$$= \frac{6}{20} - \frac{5}{20} = \frac{1}{20}$$

(c) $\int_0^M 6x \cdot (1-x) dx = \frac{1}{2}$

$6x - 6x^2$

$$\left[3x^2 - 2x^3 \right]_0^M = \frac{1}{2}$$

$$3M^2 - 2M^3 = \frac{1}{2}$$



Mode : $x = \frac{1}{2}$

$$f(x) = 6x - 6x^2$$

$$f'(x) = 6 - 12x = 0$$

$$x = \frac{1}{2}$$

HW qing 10/5

① Find the mean of the pdf given by

$$f(x) = \begin{cases} \frac{6}{125} x(5-x), & 0 \leq x \leq 5 \\ 0, & \text{elsewhere} \end{cases}$$

② Find the variance.

PdFs over infinitely long intervals

EX. $f(x)$ is a pdf:

$$f(x) = \begin{cases} 0, & x < 1 \\ \frac{2}{x^3}, & x \geq 1 \end{cases}$$

$$\frac{2}{x^3} = 2x^{-3} \\ -x^{-2}$$

(a) verify that this a pdf

$$\int_1^{\infty} \frac{2}{x^3} dx = \left[\frac{1}{x^2} \right]_1^{\infty}$$

$$= \lim_{b \rightarrow \infty} \left[\frac{-1}{x^2} \right]_1^b = \lim_{b \rightarrow \infty} \left[\frac{-1}{b^2} + 1 \right] = 1$$

(b) Find the mean ($E(X)$)

$$E(X) = \int_1^{\infty} \frac{2x}{x^3} dx = \int_1^{\infty} \frac{2}{x^2} dx$$

\nearrow $2x^{-2}$
 $-2x^{-1}$

$$= \left[\frac{-2}{x} \right]_1^{\infty} = 0 - \frac{-2}{1} = 2$$

(c) Find the median.

$$\textcircled{a} \int_1^M \frac{2}{x^3} dx = \frac{1}{2}$$

$$\left[\frac{-1}{x^2} \right]_1^M = \frac{1}{2}$$

$$-\frac{1}{M^2} + 1 = \frac{1}{2}$$

$$-\frac{1}{M^2} = -\frac{1}{2}$$

$$M^2 = 2$$

Median:

$$M = \sqrt{2}$$

HW

10 J # 3, 5

① $f(x) = \frac{1}{e^x}, x \geq 0$

(a) verify

(b) Find $E(x)$

(c) Find σ^2

(d) Find the median

$$\int -e^{-x} dx$$

$$u = -x$$

$$du = -dx$$

$$-\int e^u du$$

② $f(x) = \frac{1}{\pi} \cdot \frac{1}{x^2 + 1}, -\infty < x < \infty$

(a) verify