Probability Distributions

(for a Random Variable)

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Discrete

Roll 2 4-sided dice & add the outcomes

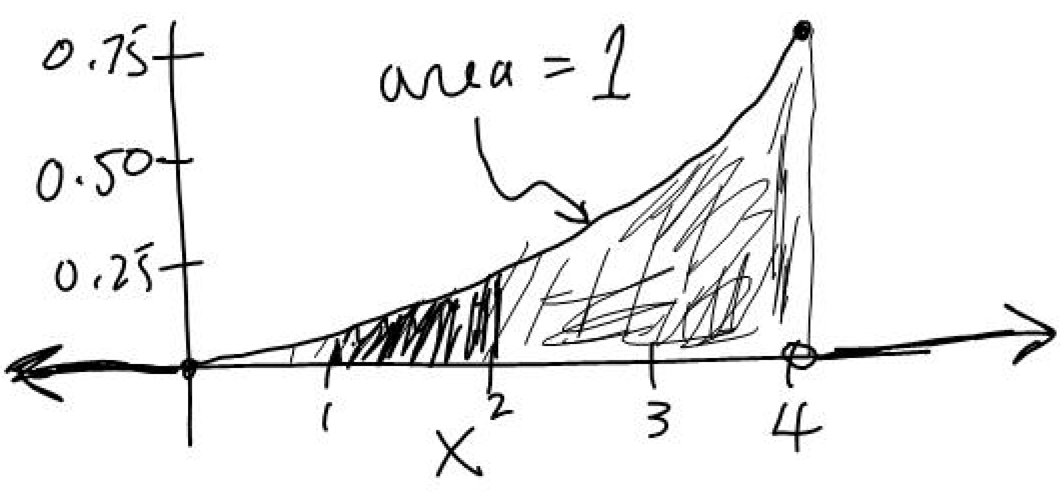
X	2	3	4	5	6	7	8
$P(X)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

$$\sum x \cdot P(x) = E(X): \frac{1}{16}(2) + \frac{1}{8}(3) + \frac{3}{16}(4) + \frac{1}{4}(5) + \frac{3}{16}(6) + \frac{1}{8}(7) + \frac{1}{16}(8) = 5$$

Continuous

The Random variable X takes on values in the interval $0 \le X \le 4$.

$$P(X = x) = \begin{cases} 0, & x < 0 \\ \frac{3x^2}{64}, & 0 \le x \le 4 \\ 0, & x > 4 \end{cases}$$



Ex – Find $P(1 \le X \le 2)$

$$P(1 \le X \le 2) = \int_1^2 \frac{3x^2}{64}\,dx$$

$$= \left[\frac{1}{64}x^3\right]_1^2 = \frac{1}{64}(2)^3 - \frac{1}{64}(1)^3$$

$$= \frac{7}{64}$$

- increase the power by 1
- divide by the new power

Ex. $P(X = 2.6) = P(2.55 \le X < 2.65)$

$<$ or $\le$

Margaret Rose was here

Ex. A continuous probability function is given by

$$f(x) = \begin{cases} k \cdot \sin^2 x, & 0 \le x \le \pi \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find k. $\displaystyle\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$$k \int_0^{\pi} \cancel{k} \cdot \sin^2 x \, dx = 1$$

$$k \left[\frac{1}{2}x - \frac{1}{4}\sin 2x \right]_0^{\pi} = 1$$

$$k \left[\frac{\pi}{2} \right] = 1 \rightarrow \boxed{k = \frac{2}{\pi}}$$

Mean of a distribution

$$\mu = \underbrace{E(X)}_{\text{expected value of } X} = \int_{-\infty}^{\infty} X \cdot f(x)\, dx$$

$$E(X^2) = \int_{-\infty}^{\infty} X^2 \cdot f(x)\, dx$$

Variance: (1st version)

$$\sigma^2 = E(X^2) - [E(X)]^2 \leftarrow$$

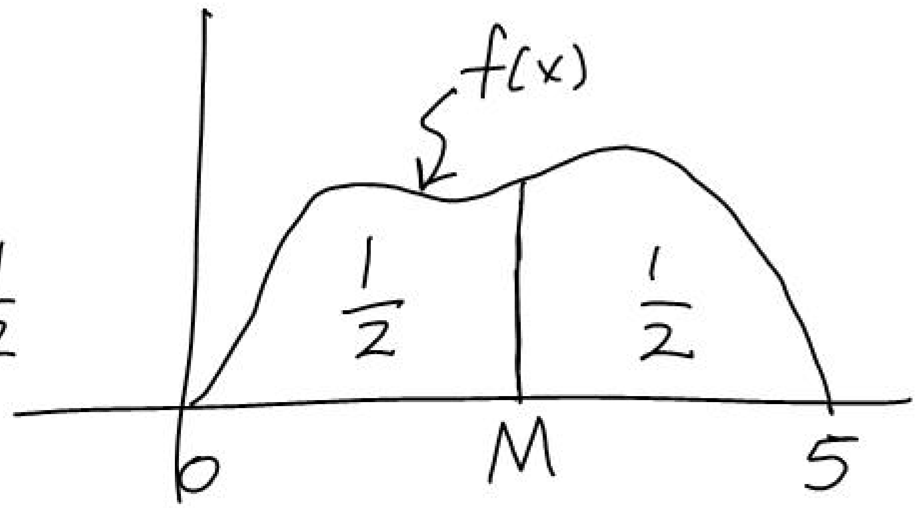
$$\sigma^2 = \int_{-\infty}^{\infty} X^2 f(x)\, dx - \left[\int_{-\infty}^{\infty} X \cdot f(x)\, dx \right]^2$$

variance (2nd version)

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\, dx$$

Median

$$\int_0^M f(x)\, dx = \frac{1}{2}$$



solve for M

EX. Find the median of the distr. with $f(x) = \begin{cases} \frac{3}{64}x^2, & 0 \le x \le 4 \\ 0 & \text{elsewhere} \end{cases}$

$$\int_0^M \frac{3}{64}(x^2)\, dx = \frac{1}{2}$$

$$\left[\frac{x^3}{64}\right]_0^M = \frac{1}{2}$$

$$\frac{M^3}{64} = \frac{1}{2} \rightarrow M^3 = 32 \rightarrow M = \sqrt[3]{32}$$

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