① $2ty\frac{dy}{dt} = t^2+1$

$$\int y\,dy = \frac{1}{2}\int \frac{t^2+1}{\cancel{2}t}\,dt$$

$$\frac{1}{2}y^2 = \frac{1}{2}\int t\,dt + \frac{1}{2}\int \frac{1}{t}\,dt$$

$$\frac{1}{2}y^2 = \frac{1}{4}t^2 + \frac{1}{2}\ln|t| + C_1$$

$$y^2 = \frac{1}{2}t^2 + \ln|t| + C$$

gen. sol. → $y = \pm\sqrt{\frac{1}{2}t^2 + \ln|t| + C}$

(b) $y(1) = -2$

$$-2 = -\sqrt{\frac{1}{2}(1)^2 \cancel{+\ln 1} + C}$$

$$-2 = -\sqrt{\frac{1}{2} + C} \rightarrow C = \frac{7}{2}$$

$$\boxed{y = -\sqrt{\frac{1}{2}t^2 + \ln|t| + \frac{7}{2}}}$$

(c)

(1, 2)

$$y = +\sqrt{\frac{1}{2}t^2 + \ln|t| + \frac{7}{2}}$$

(b) $y(4) = -\sqrt{\frac{1}{2}(4)^2 + \ln 4 + \frac{7}{2}}$

$$= -\sqrt{\frac{23}{2} + \ln 4}$$

(c) $y(4) = +\sqrt{\frac{23}{2} + \ln 4}$

#3

$$y\frac{dy}{dx} = x$$

$$\int y\,dy = \int x\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C_1$$

$$y^2 = x^2 + C$$

$$\boxed{y^2 - x^2 = C}$$

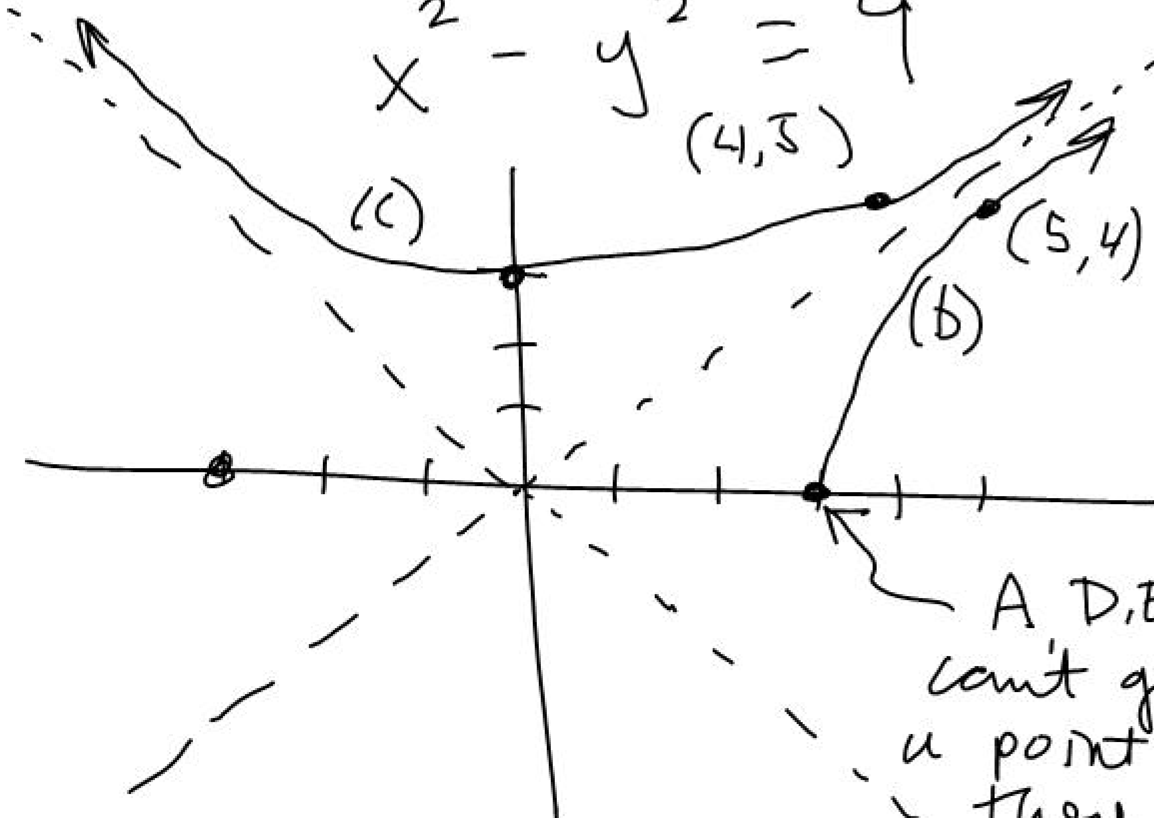
A hyperbola

(b) $(5,4)$ $\quad 4^2 - 5^2 = C$

$$-9 = C$$

$$y^2 - x^2 = -9$$

$$x^2 - y^2 = 9$$




A D.E. solution can't go through a point where there is no slope

Asymptotes

$$x^2 - y^2 = 0$$

$$y^2 = x^2$$

$$\boxed{y = \pm x}$$

$$5^2 - 4^2 = C = 9$$

$$\boxed{y^2 - x^2 = 9}$$

~~#~~4. $\displaystyle\int \frac{dP}{P^2} = \int \frac{1}{t}\,dt$

$P^{-2}$

$\frac{P^{-1}}{-1}$

$$-\frac{1}{P} = \ln|t| + C$$

$$\boxed{P = \frac{-1}{\ln|t| + C}}$$

b) $(e,-2)$ $\quad +2 = \dfrac{+1}{\cancel{\ln e}^{1} + C}$

$$\frac{1}{2} = 1 + C \longrightarrow C = -\frac{1}{2}$$

$$\boxed{g(t) = \frac{-1}{\ln|t| - \frac{1}{2}}}$$

$$g(e^2) = \frac{-1}{2 - \frac{1}{2}} = -\frac{2}{3}$$

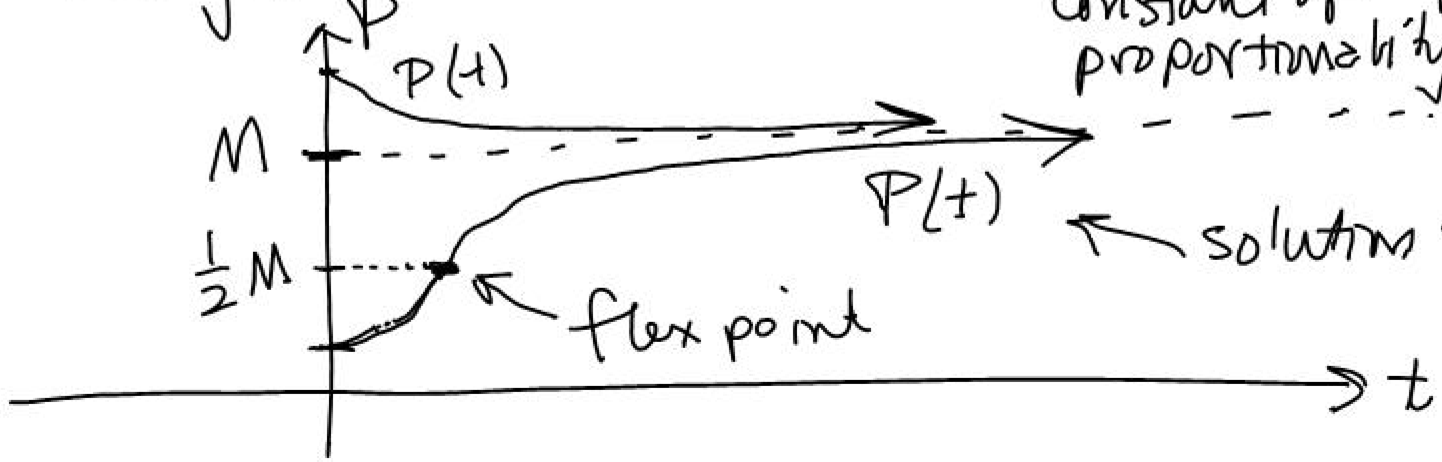
HW : #2

Logistic Growth

Limiting factor

$$\frac{dP}{dt} = kP(M-P)$$

constant of proportionality




solutions for

## Linear Differential Equations

$$\frac{dy}{dt} + P(t) \cdot y = Q(t)$$

Ex. $\frac{dy}{dx} + 3x^2 y = 6x^2$

Calculate $I(x) = e^{\int P(x)\,dx}$
the integrating factor — multiply both sides by $I(x)$

$$I(x) = e^{\int 3x^2 dx} = e^{x^3}$$

$$\underbrace{e^{x^3}\cdot \frac{dy}{dx} + e^{x^3}\cdot 3x^2 y} = e^{x^3}\cdot 6x^2$$

1st D 2nd D 1st · 2nd

Derivative of $I(x)\cdot y$ $\longrightarrow$ $\int \frac{d}{dx}\left(e^{x^3}\cdot y\right) = 2\int e^{x^3}\cdot \cancel{6}\,3x^2\,dx$

$u = x^3$

$du = 3x^2\,dx$

$\int dx = x$

$$e^{x^3}\cdot y = 2\int e^u\,du$$

$$e^{x^3} y = 2e^{x^3} + C$$

$$\boxed{y = 2 + \frac{C}{e^{x^3}}}$$

Ex. $\frac{dy}{dt} + 2ty = 1$

$$I(x) = e^{\int 2t\,dt} = e^{t^2}$$

$$e^{t^2} \cdot y = \int e^{t^2}\,dt$$

$$y = \frac{1}{e^{t^2}} \cdot \int e^{t^2}\,dt$$

$$e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots$$

$$e^{t^2} = 1 + t^2 + \frac{t^4}{2!} + \frac{t^6}{3!} + \frac{t^8}{4!} + \cdots$$

$$\int e^{t^2}\,dt = C + t + \frac{t^3}{3} + \frac{t^5}{5 \cdot 2!} + \frac{t^7}{7 \cdot 3!} + \cdots$$

$$y = \frac{1}{e^{t^2}}\left(C + t + \frac{t^3}{3} + \frac{t^5}{5 \cdot 2!} + \cdots\right)$$

for $t \approx 0$

Ex. $y' = 5y + x$

$$y' - 5y = x$$

$$I(x) = e^{\int -5\,dx} = e^{-5x}$$

$$e^{-5x} y = \int x \cdot e^{-5x}\,dx$$

$$u = x \qquad v = -\frac{1}{5}e^{-5x}$$

$$du = dx \qquad \int dv = \int e^{-5x}\,dx$$

$$e^{-5x} y = -\frac{1}{5} x e^{-5x} + \int +\frac{1}{5} e^{-5x}\,dx$$

$$e^{-5x} y = -\frac{1}{5} x e^{-5x} - \frac{1}{25} e^{-5x} + C$$

$$\boxed{y = -\frac{1}{5}x - \frac{1}{25} + Ce^{5x}}$$

Parts

$$\boxed{\int u \cdot dv = u \cdot v - \int v \cdot du}$$

~~HW~~ D.E. #2

① $\int x \cdot e^x \, dx$ ② $\int x \cdot \ln x \, dx$

③ $\int x^2 \cdot \cos x \, dx$ ④ $xy' - 2y = x^2$

⑤ $x^2 y' + 2xy = \cos^2 x$

⑥ $t \frac{dy}{dt} + 2y = t^3$, $y(1) = 0$